YANGON INSTITUTE OF ECONOMICS

Ph. D. PROGREAMME

# ARIMA MODELING FOR SELECTED ECONOMIC TIME SERIES WITH OUTLIERS

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JUNE, 2010

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JUNE, 2010

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Partial fulfillment of the requirement for the degree of

Ph. D.

of the Department of Statistics

Yangon Institute of Economics

Submitted by

Mya Thandar

June, 2010

# Certification

I hereby certify that the content of this thesis is wholly my own work unless otherwise referenced or acknowledged. Information from sources is referenced with original comments and ideas from writer him/herself.

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# ARIMA MODELING FOR SELECTED ECONOMIC TIME SERIES WITH OUTLIERS

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# ABSTRACT

Nontypical values in a data set are called outliers. The effects of outliers on a time series may be serious because of dependence structure of successive observations. Therefore, it is important to detect and handle outliers before the analysis of a time series. This study attempts to investigate the effects of additive outlier (AO) and innovational outlier (IO) in a time series using simulation data, and to demostrate the detection and identification of outliers in selected economic time series of Myanmar. From the simulated results, it is observed that the effect of IO on mean is more significant for both AR(1) and ARMA(1,1) series but that of AO on the mean is more prominent for MA(1) series. In addition, the effect of IO on the variance is more obvious than that of AO for AR(1), MA(1) and ARMA(1,1) series. The simulated results suggest that the percentage of correct model selection declines as the value of outlier increases in the AO whereas it is not true in the IO of both AR(1) and MA(1) series. Outliers can frequently occurred in economic time series due to unusual events. Since the govenrment policy or implementation of new rule and regulation had changed in our country, many economic time series could be affected. In the detection and identification of outliers, there observed AR(1) model with an IO outlier in 1986-87 and two AO's in 1994-95 and 1998-99 for base metal and ores export series, AR(1) model with an AO in 1978-79 for teak export series, AR(1) model with an AO in 1978-79 and an IO in 1983-84 for wheat production series, AR(1) model with an AO in 1979-80 for lablab bean production series and AR(1) model with an AO in 1963-64 for lima bean production series. Based on the outliers detected, the most fitted model of each series is constructed for forecasting purpose.

# ACKNOWLEDGEMENTS

My deepest appreciation and gratitude goes to the following who served as the source of my strength and confidence and became reasons for my persistence in pursuing this thesis:

To Professor Dr. Kan Zaw, Rector, Yangon Institute of Economics, for his kind permission to write this thesis and for his guidance to submit this thesis;

To Professor U Kyaw Min Htun, Pro-Rector, Yangon Institute of Economics, for his invaluable comments and suggestions regarding this thesis;

To Professor Dr. Soe Win, Acting Director General, Department of Higher Education (Upper Myanmar), for his generous guidance in every phase of this thesis, for his help in the conceptualization and implementation of this thesis, for his insight comments and suggestions to complete my thesis successfully;

To Professor Dr. Lay Kyi, Head, Department of Statistics, Yangon Institute of Economics, for his helpful guidance and useful suggestions to accomplish this thesis;

To my supervisor Professor Dr. San Kyi, Department of Statistics, Yangon Institute of Economics, for her consideration and encouragement, for counseling and giving precious advice for this thesis, and for her kind support in editing this thesis;

To Professor Dr. Thet Lwin, Head, Department of Statistics (Retd.), Yangon Institute of Economics, Professor Daw Khin San Myint, Head, Department of Statistics (Retd.), Yangon Institute of Economics, Professor Dr. Than Toe, Head, Department of Statistics (Retd.), Yangon Institute of Economics, for their constructive suggestions and comments, invaluable technical advice, expert opinions and generous editorial assistance in my thesis;

To Professor Dr. Sandar Oo, Head, Department of Management Studies, Yangon Institute of Economics, Professor Dr. Soe Soe Hlaing, Head, Department of Mathematics, Yangon Institute of Economics and Professor Dr. Thaung Htay, Head, Department of Statistics, Monywa Institute of Economics for contributing their invaluable comments and suggestions to improve this thesis;

To all my beloved and honourable teachers since my childhood up to now, for educating me and to those who helped me in one way another for the completion of my thesis;

To staffs of the Yangon Institute of Economics Library, for their hospitality and cordial help to access primary documents and other sources of data and information;

To my parents, for their lasting support and for having been serving as my inspiration and to my sister and brother, for their love and indulgent on all aspects of my life.

# TABLE OF CONTENTS

# LIST OF TABLES

## LIST OF FIGURES

# LIST OF ABBREVATIONS

| | |
|---|---|
| ACF | Autocorrelation Function |
| ADV | Adjustment Diagnostic based on Variance estimate |
| AIC | Akaike's Information Criterion |
| AO | Additive Outlier |
| AR | Autoregressive |
| ARMA | Autoregressive Moving Average |
| ARIMA | Autoregressive Integrated Moving Average |
| BIC | Bayesian Information Criterion |
| CLS | Conditional Least Squares |
| DC | Diagnostics for Coefficient |
| DV | Diagnostics for the Innovation Variance |
| GM-estimate | Generalized Maximum Likelihood-estimate |
| IO | Innovational Outlier |
| LD | Likelihood Displacement |
| LRT | Likelihood Ratio Test |
| LS | Level Shift |
| MA | Moving Average |
| MAD | Mean Absolute Deviation |
| MAPE | Mean Absolute Percentage Error |
| MCMC | Markov Chain Monte Carlo |
| ME | Mean of Error |
| ML | Maximum Likelihood |
| MLE | Maximum Likelihood Estimate |
| MPE | Mean Percentage Error |
| MSE | Mean Squared Errors |
| OLS | Ordinary Least Squares |
| PACF | Partial Autocorrelation Function |
| PE | Percentage Error |
| RC | Relative Change |
| RO | Reallocation Outlier |
| RSE | Residual Standard Error |
| RSS | Residuals Sum of Squares |
| SE | Standard Error |
| SPSS | Statistical Package for Social Sciences |
| SSE | Sum of Squares Errors |
| STDS | Statistical Time Series Diagnostic Software |
| TC | Temporary Change |
| VC | Variance Change |

# CHAPTER I

# INTRODUCTION

## 1.1 Rationale of the Study

A time series is an ordered sequence of observations on a variable of interest collected usually in time, particularly in terms of some equally spaced time intervals. Time series exist in several fields such as agriculture, business, quality control, economics, engineering, geophysics, medical studies, meteorology, natural sciences and social sciences.

There are many different objectives for examining time series. These include the understanding and description of the generating mechanism, the forecasting of future values, and optimal control system. The main part of statistical methodology available for analyzing time series is referred to as time series analysis. An important feature of a time series is that, typically, adjacent observations are dependent or correlated. Because of dependence structure, statistical procedures and techniques that rely on independence assumptions are not applicable and different statistical techniques are needed for a time series analysis. It is required to develop a suitable stochastic model for the analysis of observed time series data and it can be used in several areas of application such as economics, business, engineering, industrial research, physical and social sciences, etc.

Economic time series are sometimes influenced by special events or circumstances such as political or economic changes, strikes, outbreaks of war, monetary crises, implementation of a new rule and regulation, advertising promotions and similar events. These events are referred to as intervention events and they usually bring outliers into the time series data.

Direct use of conventional statistical time series analysis may occasionally ignore the fact that the observed time series no longer cover the time period with the same condition. Consequently, it leads to use the inadequate model and to have the biased estimates of the parameters in time series analysis. The intervention analysis can be used to analyze the time series with outliers when the timing of intervention is known. The method is then generalized to study the impact of the events when the timing of interventions is unknown and hence leads to the general time series outlier analysis.

Time series with outlying observations needs to be analyzed using statistical outlier analysis. The effect of the change due to the unusual event and it's position in time series should be analyzed in order to provide the most suitable forecasts for the future values. Thus the investigation into the presence of outliers, identification of outliers, assessment of their effects and the remedial measures to accommodate the outliers become a crucial aspect of analyzing many economic time series and have gained much important.

A time series with outliers can be model by adding intervention into ARIMA. An ARIMA (Autoregressive Integrated Moving Average) model is an agebraic statement telling how observations on available are statistically related to past observations on the same variable. ARIMA models has three advantages over many other traditional univariate time series models. First, the concepts associated with ARIMA models are derived from a solid foundation of classical probability theory and mathematical statistics. Second, ARIMA models are a family of models, not just a single model, more appropriate models can be chosen out of this larger family of models. Third, an appropriate ARIMA model produces optimal forecasts, that is, no other single-series model can give forecasts with a smaller mean sequred forecast error. Thus, ARIMA models can handle a wider variety of situations and provide more accurate short-term forecasts than any other single-series technique.

In Myanmar, a change in the government policy (for example, from the centralized economy to market oriented economy system) as well as implementation of a new rule and regulation can be taken as an intervention, which can change the level of several economic time series. Since the economic policies had changed to get a better situation in our country, many time series data could be affected by the policy changes.

Hence, this study would make attempts to detect and distinguish the outliers in selected economic time series using the available outlier detection methods. Moreover, possible reasons for the presence of the identified outliers in observed data series will also be investigated. Then, the ARIMA model with outliers will be constructed for the selected economic time series. The fitted models will be applied to forecast the future values, which can be used in decision making and policy making.

## 1.2 Method of Study

An analytical method with the support of tables, figures, graphs and plots has been extensively used in this study. This method is observed to be more suitable to the nature and characteristics of the observed data series. More emphasis is put on analytical method of time series analysis and forecasting for the analysis of the selected economic time series data.

Moreover, the simulations are carried out in order to study the effects of outliers on various statistics like mean, variance, some model identification tools such as autocorrelation function (ACF) and partial autocorrelation function (PACF) as well as some model selection criteria including Akaike's Information Criterion (AIC) and Bayesian Information Criterion (BIC).

The data used in this study are compiled from various annual publications of Report to the People, Report to the Pyithu Hlyuttaw, Review of Financial, Economic and Social Conditions, Statistical Yearbook and various publications of Selected Monthly Economic Indicators from 1950 to 2008.

## 1.3 Objectives of the Study

The objectives of the study are as follows:

(1) To review various outlier detection methods and their advantages and disadvantages

(2) To investigate the effects of outliers in model identification and parameter estimation

(3) To construct ARIMA models for selected economic time series with outliers and

(4) To identify the positions and to distinguish the types of detected outliers.

## 1.4 Scope and Limitations of the Study

In this study, the nature and characteristics of the selected economic time series are pointed out by using the basic statistics like mean, variance, autocorrelation function (ACF) and partial autocorrelation function (PACF) etc.

Only the stochastic univariate time series model, namely Autoregressive Integrated Moving Average (ARIMA) model is considered since most of the outlier detection methods and techniques have been applied to ARIMA model. Existing methods for detection and identification of outliers are used in ARIMA model fitting to selected economic time series.

Outlier problems considered in this study are tackled only in time domain. Only non-seasonal ARIMA time series models are considered in this study.

Among the various types of outliers which can occur in a time series, only additive outlier (AO) and innovational outlier (IO) are considered most often in the literature and this study focuses on these two types of outliers.

## 1.5 Organization of the Study

This study has been outlined in the following manners.

Chapter I mentions rationale of the study, method of study, objectives of the study, scope and limitations of the study as well as organization of the study.

Chapter II introduces definition and types of outliers and reviews the possible effects of outliers in time series, ARIMA models for time series with outliers, effects of an outlier on original and error series and related literature as well as some procedures for detection of outliers in time series together with their advantages and disadvantages.

Chapter III explores the effects of outlier in model identification and parameter estimation using simulated data.

Chapter IV includes ARIMA modeling for detection and estimation of outliers in selected economic time series based on selected outlier detection methods.

Chapter V is concerned with forecasting and forecast evaluation for selected economic time series with outliers.

Final chapter highlights conclusion, suggestions and further research problems in the case of ARIMA modeling with outliers.

# CHAPTER II

# REVIEW OF OUTLIERS IN TIME SERIES DATA

## 2.1 Definition and Types of Outliers

Outliers in a time series data set can occur for different reasons. There are two types of anomalies, namely gross errors and outliers. Gross errors are faulty observations, for example, measurement, reading and typing errors. Identifying these is the least controversial aspect of outlier detection, since gross errors should naturally be identified and corrected whenever possible.

Time series data may often be affected by unusual events, external disturbances or errors which create spurious observations that are inconsistent with the rest of the series. When the timings of such unusual observations are unknown, they are referred to as outliers if the events have large impact on the time series. If the observation treated as a potential outlier cannot be shown to be a gross error, then it has to be considered as an outlier. These observations, called outliers and their detection, identification and modeling are considered in this study.

Outliers can take several forms in time series. The formal definition and a classification of outliers in time series context were first proposed by Fox (1972). He proposed a classification of time series outliers to type I and type II based on an autoregressive model. These two types have later been renamed as additive and innovational outliers, and are usually abbreviated as AO and IO respectively. AO affects single observation and there is no "carry-over" effect. IO affects the observations from the outlier position onwards and it has "carry-over" effect as well as decays.

Usually only AOs and IOs are considered in the literature, but Tsay (1988) defines three more types of outliers, namely level shifts (LS), temporary or transient changes (TC) and variance changes (VC). These LS and TC are not strictly speaking outliers, but rather structural changes. The LS (sometimes known as level change), simply changes the level (or mean) of the series by a certain magnitude from a certain observation onwards. A TC is a generalization of AO and LS in the sense that it causes an initial impact like an AO but the effect is not permanent and this effect dies out gradually as exponentially decays. In some models, the effect of TC will be very close to the effect of IO. A VC simply changes the variance of the observed series by adding a new zero mean random variable and it does not affect the level of the series.

Furthermore, Wu, Hosking and Ravishanker (1993) proposed an interesting type; a reallocation outlier (RO) that is defined as a batch of AO, the sum of whose effects is equal to zero.

## 2.2 ARIMA Models for Time Series with Outliers

In the outlier literature (e.g. Tsay, 1986 and 1988; Chen and Liu, 1993), a time series with outliers is modeled as ARIMA plus intervention. The basic reference to ARIMA model is Box and Jenkins (1976).

The parametric approach to modeling a time series in terms of linear difference equations has led to an important class of models, namely autoregressive integrated moving average model with order p, d and q, popularly known as ARIMA (p, d, q) (Box and Jenkins, 1976).

If $Z_t$ is an observed time series, then the ARIMA (p, d, q) model is

$$\phi (B) (1-B)^d Z_t = \theta (B) a_t \qquad (2.2.1)$$

where $\phi (B) = 1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p$ and $\theta (B) = 1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q$ are polynomials of degree p and q in B, $\phi_i$, i = 1, 2, . . . , p and $\theta_j$, j = 1, 2, . . . , q are the autoregressive and moving average parameters of the time series respectively and B is the backward shift operator, that is, $B^j Z_t = Z_{t-j}$ . In the above model, $a_t$ is the white noise or error series with mean zero and variance $\sigma_a^2$ referred to as the error variance.

It is assumed that the series $(1-B)^d Z_t$ to be stationary, i. e., the roots of $\phi (B) = 0$ lie outside the unit circle, and invertible, that is, the roots of $\theta (B) = 0$ lie outside the unit circle. When d = 0, Equation (2.2.1) represents a stationary process ARMA (p, q), given by

$$\phi (B) Z_t = \theta (B) a_t . \qquad (2.2.2)$$

The ARMA (p, q) process $Z_t$ can also be represented as a random shock model of the form

$$Z_t = \psi (B) a_t \qquad (2.2.3)$$

where $\psi(B) = 1 + \psi_1 B + \psi_2 B^2 + \ldots$ and $\psi$ weights are calculated by equating the coefficients of B in the equation $\phi(B)\,\psi(B) = \theta(B)$. For the series to be stationary, it is assumed that $\psi(B)$ converges for $|B| \leq 1$, that is, on or within the unit circle. Alternatively, the $\psi$ weights have the condition $\sum_{j=0}^{\infty}|\psi_j| < \infty$. Similarly, $Z_t$ can also be represented as an inverted form of the model using the $\pi$ weight as

$$\pi(B)\,Z_t = a_t \tag{2.2.4}$$

where $\pi(B) = 1 - \pi_1 B - \pi_2 B^2 \ldots$. The $\pi$ weights are analogously obtained by equating coefficients of B in $\phi(B) = \theta(B)\,\pi(B)$. To satisfy the condition of invertibility it is assumed that $\pi(B)$ converges on or within the unit circle. Alternatively, the $\pi$ weights are assumed to satisfy the condition $\sum_{j=0}^{\infty}|\pi_j| < \infty$. (Box, Jenkins and Reinsel, 1994).

Following Box and Jenkins (1976), analysis based on these models has been extensively studied in the literature and for details Abraham and Ledolter (1983), Chatfield (1989), Kendal and Ord (1990), Wei (1990), Box et al. (1994), Mills (1994), Brockwell and Davis (1991, 1996) and Liu (2006) are referred.

Box et al., (1994) suggested that the principle of parsimony is important in model building that is, the number of parameters p, d, and q of the fitted model must be minimum. The inferential problems considered in the literature are identification of the order p, d, and q in the model, estimation of the time series parameters and error variance, diagnostic checking of the model, and forecasting the future values, etc.

In this study, the analysis of stationary and invertible time series ARMA (p, q) with outliers are considered.

Let $Y_t$ be the observed time series and $Z_t$ be the underlying time series which is free of the impact of outliers. Assume that $Z_t$ follows a general ARIMA model in Equation (2.2.1). Then the general outlier model for on observed time series $Y_t$ is defined as

$$Y_t = f(t) + Z_t \tag{2.2.5}$$

where $Z_t$ is a regular ARIMA model and outliers are incorporated through f(t). The f(t) can take different outliers types.

An additive outlier (AO) model, that is, $f(t) = \omega P_t^{(T)}$ at time T in ARMA (p, q) (Fox, 1972; Abraham and Box , 1979) is

$$Y_t = \omega P_t^{(T)} + Z_t \tag{2.2.6}$$

where $Y_t$ is the observed series, $Z_t$ is an unobserved outlier free series as in Equation (2.2.1), $\omega$ is the outlier parameter $-\infty < \omega < \infty$ and

$$\begin{aligned} P_t^{(T)} &= 1, \quad t = T, \\ &= 0, \quad t \neq T, \end{aligned}$$

is the indicator variable representing the presence or absence of an outlier at time T. The presence of AO is almost clearly seen in a time sequence plot as AO does not have any carry- over effect.

An innovational outlier (IO) model at time T, that is, $f(t) = \omega \psi(B) P_t^{(T)}$, in ARMA (p, q) is specified by (Fox 1972; Abraham and Box, 1979)

$$Y_t = \omega \psi(B) P_t^{(T)} + Z_t \tag{2.2.7}$$

where $Y_t$ is the observed series, $Z_t$ is an unobserved outlier free series as in Equation (2.2.1), $\omega$ is the outlier parameter $-\infty < \omega < \infty$ and

$$\begin{aligned} P_t^{(T)} &= 1, \quad t = T, \\ &= 0, \quad t \neq T, \end{aligned}$$

is the indicator variable which represents the presence or absence of an outlier at time T. The IO affects all observations $Y_T$, $Y_{T+1}$, . . . beyond time T and decays with $\psi$ weights as it has carry-over effect.

In addition to these two main outlier types, two other types of outlier models were proposed in the literature to handle sudden level changes which may be temporary or permanent type, called the temporary change (TC) model and level shift (LS) model (Tsay, 1988; Chen and Liu, 1993) respectively.

The TC model at time T, that is, $f(t) = \dfrac{\omega}{(1-\delta B)} P_t^{(T)}$, is expressed as

$$Y_t = \frac{\omega}{(1-\delta B)} P_t^{(T)} + Z_t \qquad (2.2.8)$$

where $\delta$ is the damping factor with $0 < \delta < 1$. A TC causes an initial impact on the observation at time T like an AO but the effect passes on to the following observations. The impact of a TC is not permanent; it decays exponentially according to some dampening factor, $\delta$. If $\delta = 0$, the TC model is the same as the AO model in Equation (2.2.6).

The LS model at time T, that is, $f(t) = \frac{\omega}{(1-B)} P_t^{(T)}$, is the same as Equation (2.2.8) when $\delta = 1$, given by

$$Y_t = \frac{\omega}{(1-B)} P_t^{(T)} + Z_t \qquad (2.2.9)$$

where $\omega$ can be either positive or negative and the change is permanent . A LS simply changes the level (or mean) of the series by a constant magnitude $\omega$ from a certain observation $Y_T$ onwards till the end of the series.

It is not unusual to come across time series data with more than one outlier. The problem of handling multiple outliers in time series is more complicated, for the simple reason that the outliers could be of different types (Barnett and Lewis, 1994).

More generally, an observed time series $Y_t$ might be affected by outliers of different types at several points of time $T_1, T_2, \ldots, T_k$ and we have the following multiple outliers model of the general form

$$Y_t = \sum_{j=1}^{k} \omega_j V_j(B) P_t^{(Tj)} + Z_t \qquad (2.2.10)$$

where k is the total number of outliers present in the series, $\omega_j$, $j = 1, 2, \ldots, k$ are the corresponding outlier parameters which may not be distinct and

$$\begin{aligned} V_j(B) &= 1 && \text{for an AO,} \\ &= \psi(B) && \text{for an IO,} \\ &= \frac{1}{(1-\delta B)} && \text{for a TC,} \\ &= \frac{1}{(1-B)} && \text{for a LS.} \end{aligned}$$

when an outlier type present at time point $T_j$, $j = 1, 2, \ldots, k$.

Another outlier type, variance change (VC) is still further away from the AO and IO outlier types, and is not usually considered in connection with outliers at all . It does not affect the level of the series directly like the other types of outliers. A VC simply changes the variance of the observed series at a time by adding a zero mean, variance $\omega\sigma_a^2$ random variable to the observed series. This changes the observed variance from $\sigma_a^2$ to $(1+\omega)^2\sigma_a^2$.

Wu et al.(1993) proposed an interesting further type, a reallocation outlier (RO). This is defined as a batch of additive outliers, the total sum of whose effects is equal to zero. A RO model is defined as

$$Y_t = \sum_{j=0}^{k} \omega_j P_t^{(T)} + Z_t \qquad (2.2.11)$$

with the restriction that the sum $\Sigma\,\omega_j$ be equal to zero. In this formulation there are $k+1$ outliers of magnitude $\omega_j$ at time $t = T, T+1, T+2, \ldots, T+k$.

The problems associated with these types of outlier models are to identify the timing and the type of outliers and to estimate the magnitude $\omega$ of the outlier effect. Among the various types of outliers which can occur in a time series, the AO and IO are considered most often in the literature and only these two types of outliers are focused in this study.

## 2. 3 Effects of an Outlier on Original and Error Series

In this section, the effect of an outlier on the original series (outlier free series) $Z_t$ which is a stationary and invertible ARMA (p, q) and the effect of an outlier on its error series $a_t$ where $t = 1, 2, \ldots, n$ are presented. Consider the observed series $Y_t$, contaminated by a single outlier of AO or IO, given by

$$\begin{aligned} Y_t &= \omega P_t^{(T)} + Z_t, && \text{for AO} \\ &= \omega\psi(B)P_t^{(T)} + Z_t, && \text{for IO.} \end{aligned} \qquad (2.3.1)$$

The model in Equation (2.3.1) allows the effect on only a single observation at time point $t = T$ of the original series $Z_t$ to be affected by AO, whereas the effect of IO at $t = T$ carries over to the subsequent observations of $Z_t$ and decays with $\psi$ weights.

Suppose the observed series $Y_t$ is treated as a typical stationary and invertible ARMA (p, q) series ignoring the presence of outliers. Then the residual series of $Y_t$ is defined as

$$e_t = \pi(B)Y_t \qquad (2.3.2)$$

where $\pi(B) = \psi^{-1}(B) = 1 - \sum_{i=1}^{\infty} \pi_i B^i$.

The error series of $Z_t$ is given by $a_t$ where

$$a_t = \pi(B)Z_t. \qquad (2.3.3)$$

However, since $Y_t$ is an outlier contaminated series, using Equations (2.3.1) and (2.3.3), $e_t$ can be expressed in terms of $a_t$ as

For AO, $e_t = \pi(B)Y_t = \pi(B)[\omega P_t^{(T)} + Z_t]$

$= \omega\pi(B)P_t^{(T)} + a_t$

For IO, $e_t = \pi(B)Y_t = \pi(B)[\omega\psi(B)P_t^{(T)} + Z_t]$

$$= \omega P_t^{(T)} + a_t \qquad (2.3.4)$$

From Equation (2.3.4), it can be seen that the presence of IO affects a single observation of $a_t$ series at only one time point t = T, whereas the effect of AO on $a_t$ carries over to the subsequent observations and decays with $\pi$ weights (Chang et al, 1988; Box et al, 1994).

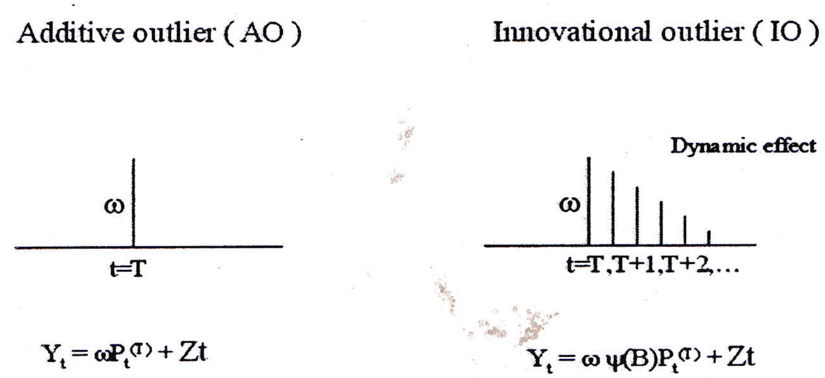
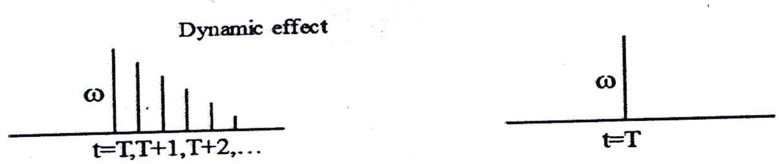



**Figure 2. 1 The Effect of an Outlier on $Z_t$ Series**

Figures 2.1 and 2.2 were presented by Soe Win (2004) to state a hypothetical situation representing the effect of outlier of either type on original series $Z_t$ and error series $a_t$ respectively with the outlier parameter of magnitude $\omega > 0$. Note that the dynamic effect of IO on observations and that of AO on error series can be either positive or negative depending on the signs of the weights $\psi$, $\pi$, and $\omega$.

Additive outlier ( AO ) Innovational outlier ( IO )




$e_t = \omega\ \pi(B) P_t^{(T)} + a_t$ $e_t = \omega P_t^{(T)} + a_t$

**Figure 2. 2 The Effect of an Outlier on $a_t$ Series**

To know the effect of an outlier on the analysis of the observed series, one must understand the effect on the mean and variance, the estimates of time series parameters, error variance, some other model identification tools such as autocorrelation and partial autocorrelation functions, as well as some model selection criteria including Akaike's Information Criterion (AIC) and Bayesian Information Criterion (BIC).

## 2.4 Possible Effects of Outliers in Time Series

The existence of the outliers has significant influence on the analysis of time series data. The outliers may influence adjacent observations due to the presence of autocorrelation pattern in a time series. The dependence structure of time series observation makes the detection of outliers difficult.

In time series model building, there are four stages including identification, estimation, diagnostic checking and forecasting. Thus, the existence of outliers in a time series can affect on all these stages.

The model identification in the presence of outliers in a time series will be misleading, unless outliers are somehow taken into account (Glendining, 1998). In series of short to moderate length, the presence of a single outlier will often result in a falsely model identification and the identified lag lengths will also be wrong. (Deutsch, Richards and Swain, 1990).

Tsay (1986) also noted that exact results for the effects of outliers are complicated and require lengthy computations. The outliers cause substantial biases in estimated model parameters (Chen and Liu, 1993). Various model estimation procedures such as least squares and maximum likelihood methods are both sensitive to the presence of outliers (Bustos and Yohai, 1986). Consequently, the misspecified model can be encountered in diagnostic procedure and it can lead to serious errors in forecasting.

Outliers have also fairly obvious effects on forecasts, and especially outliers near the beginning of the forecast period can have serious consequences. Point forecasts may suffer only a little effect but the prediction intervals can become severely misleading, as outliers inflate the estimated variance of the series (Ledolter, 1989 and Hotta, 1993). Some outliers have more serious effects on point forecasts even when outliers are not close to the forecast origin, and they increase the width of prediction intervals as well (Trivez, 1993).

In addition, outlier analysis has two special problems to consider, namely, smearing and masking. These concepts are related to the detection of outliers, and can even be intertwined. When one outlier affects the series in a manner that makes other observations appear to be outliers as well, where they are in fact not, it is called smearing. Conversely, masking means that one outlier hides others from being detected. These notions are closely connected to specific outlier detection methods, and are not properties of the data itself. In other words, smearing and masking are only deficiencies of certain methods, not type of outliers as such.

Apart from affecting the estimates of model parameters and forecasting, the outliers can distort the model specification itself and the impact of outliers in time series modeling can be serious enough to affect the credibility of the model (Barnett and Lewis, 1994).

For the existence of outlier's effect on the model building procedure, the conventional time series analysis may have some problems and outliers appeared by time series nature should not be deleted. Therefore, the positions of outliers and the pattern of outliers are necessary to know for constructing the models for time series with outliers.

When the position of outliers is known, an appropriate handling can be made using intervention analysis (Box and Tiao, 1965). However, the presence of outliers in many situations is rarely known beforehand. Hence the crucial step in analysis of time series with outliers is the construction of an appropriate model representing outliers in the data. In doing so, it is important to understand the nature of outliers and their impact on the time series.

## 2.5 Literature Review

### 2.5.1 Detection and Identification of Outliers

Fox (1972) introduced models to accommodate the presence of two types of outliers called additive outlier (AO) and innovational outlier (IO) in autoregressive time series data. He also proposed a likelihood ratio test for outlier detection. However, he did not propose any method to differentiate between AO and IO.

Abraham and Box (1979) were the first to introduce the Bayesian approach in the time series models with the outlier types AO and IO. Then, extended the models introduced by Fox (1972) to ARMA (p, q) and Bayesian parametric inference procedure for time series parameters as well as outlier parameters in the presence of both types of outliers in the case of AR (p) were carried out. Based on generated data from AR (1), some numerical examples were also presented by Abraham and Box.

Following Fox (1972), an iterative detection procedure for outliers in ARIMA models was proposed by Chang and Tiao (1983). This procedure is based on likelihood ratio test and involves the identification of the outlier types as well. They also showed that AO can cause serious basis in parameter estimation whereas IO only have minor effects in estimation. Tiao (1985) also illustrated this procedure using two real life data sets.

Muirhead (1986), removing the drawback of Fox's approach, introduced a likelihood ratio test for detection of single outlier and identification of the type of outlier. He proposed a Bayes rule to differentiate between AO and IO type. The proposed method for the identification of outliers was also compared with the corresponding Bayes rule proposed by Abraham and Box (1979).

A number of authors addressed the Bayesian approach to outliers in time series. As presented earlier, Abraham and Box (1979) were the first to present the Bayseian analysis which was followed by Murihead (1986), who proposed a Bayes rule to differentiate between AO and IO type. Then, Smith (1983) considered a general approach to robust Bayesian methods with specific consideration of outliers in time series. Ameen and Harrison (1985) presented Bayesian forecasting methods in the presence of outliers from contaminated sources.

All the iterative likelihood ratio test procedures were based on the assumption that time series model is known. Tsay (1986) whereas proposed an iterative procedure for model specification of time series in the presence of outliers by using effectively the iterative outliers detection procedure proposed by Chang and Tiao (1983).

The iterative likelihood ratio test procedure for detection of IO and AO and estimation of time series parameters of ARMA in the presence of outliers was also provided by Chang, Tiao and Chen (1988). In their work, a detailed discussion on the performance of the iterative procedure in the context of AR (1) and MA(1) with up to two outliers, was also made on the basis of simulation.

By using a similar iterative outlier detection and identification procedure based on likelihood ratio test, Chen and Liu (1993) introduced a procedure for joint estimation of model parameters and outlier effect in ARMA. In this procedure, if an outlier is detected at any stage of iteration, depending on the detection of outlier type, the series can be properly adjusted. It was revealed that if an IO is detected at time point, the adjusted observation at t = T is the conditional expectation of the original observation given the past. It is like the AO case, where the adjusted observation is an interpolation based on past and future observations. Chen and Liu, also after investigating the effects of different types of outliers on the observed series, expressed that " ... the effect of an IO is more intricate than the effects of other types of outliers."

### 2.5.2 Robust Estimation Procedures in the Presence of Outliers

Robust estimation procedures for parameters of time series in the presence of outliers have been considered by a number of authors. Firstly, Denby and Martin (1979) proposed a class of generalized maximum likelihood estimates (GM-estimates) for AR (1) model in the presence of a single outlier of either type. It was shown that though GM-estimate perform moderately well in the presence of outliers AO and IO, the M-estimates perform much better in the presence of IO.

Martin (1979) extended the GM-estimates to AR(p) model. He also discussed some theory and methodology of robust estimation for time series with AO and IO as well as the problem of patch outliers. Furthermore, he proposed a formal significance test and a residual plotting diagnostic technique for determining the outlier type.

Martin and Yohai (1985) presented a detailed review of robust estimators for ARMA model with outliers. In their paper, they also briefly discussed the influence curve of time series with AO and IO from isolated to patch outliers.

Bustos and Yohai (1986) pointed out that the GM-estimator has a complicated asymptotic covariance matrix. They proposed two new robust estimators based on residual autocovariances (RA-estimators) and truncated residual autocovariances (TRA-estimators). The proposed estimators were compared with least squares (LS) estimator, M and GM estimators for AR(1) and MA(1) models with AO and IO outliers. Based on Monte Carlo results, it was shown that RA estimators are not qualitativelty robust when the model has the moving average part but much stable than LS and M estimators in the presence of AO.

Luceno (1998) proposed robust estimators in the presence of nonconsecutive multiple outliers in ARMA (p, q) series based on re-weighted maximum likelihood estimator using Huber or redescending weights. A multiple outlier detection procedure was also introduced by choosing appropriate weights for robust ARMA (p, q) fitting.

### 2.5.3 Influence Function Approach to Outliers

A different approach to outlier detection based on influence function was proposed by Chernick, Downing and Pike (1982). Using the influence functions of the autocorrelation function, they investigated the effect of outliers on stationary time series but this procedure cannot identify the types of outliers.

Various diagnostic procedures for deletion of outliers and influential points in regression models have been discussed in literature (Cook and Weisberg, 1982; Chatterjee and Hadi, 1988). Abraham (1987) and Pena (1987) also were the first in adapting some of these procedures to time series models. Abraham discussed diagnostic tests based on deletion of suspected observations and in particular the impact of deletion on Q statistic (Draper and John, 1981) for AR (1) series.

Pena (1987) also discussed sample influence function for parameters in the presence of outliers in ARMA model as another attempt in this direction. By treating a single observation as missing in turn, Pena (1987) investigated the impact of outlier on parameter estimation through sample influence function. Using least squares predictors, the missing values were estimated. The procedure is a natural generalization of leave-one-out technique in regression diagnostic. A somewhat similar example of influence methods was also presented by Abraham and Chuang (1989), though only for AR models.

### 2.5.4 Deletion Diagnostics and Other Procedures

Abraham and Chuang (1989) investigated the effect of deletion of k observations on Q statistics in case of ARMA (p, q) and they proposed an outlier detection procedure based on Q statistic which can be used for identification of outlier type as well. They also presented a modal building strategy based on the investigation of the pattern of Q statistic. Abraham and Chuang (1989) also claimed that in time series analysis, some suspected outliers may lead to large residuals but may not affect the parameter estimates, whereas other outliers may not only lead to large residuals but also may affect model specification and parameter estimation.

Bruce and Martin (1989) proposed an in-depth leave-k-out diagnostics approach to outliers in time series, where k consecutive observations were treated as missing and the parameters were estimated using Kalman filter in case of ARIMA (p, d, q). They also investigated the effect of missing observations on sample influence function of time series parameters and error variance, which led to various diagnostic tests. Since k consecutive observations were treated as missing, the proposed procedure was shown to handle outlier's patch and avoid the masking effect. While investigating the usefulness of various estimators in outlier diagnostic procedure, Bruce and Martin (1989) showed that the diagnostic based on error variance has better properties than the diagnostic based on time series parameters.

Cook (1986, 1987) proposed a general measure of model perturbation using likelihood displacement measure. Ledolter (1990) also made another attempt at using deletion diagnostics for detecting outliers in time series. The impact of outlier on Cook D statistic (Cook and Weisberg; 1982), which is equivalent to likelihood displacement criterion, was investigated by treating each observation as missing. He also investigated the behaviour of the proposed diagnostic procedure and presented simulation study for AR(1) process. He further investigated the additive outlier model and claimed that the sensitivity of variance estimate depends on the difference between observations and their interpolation values, which justifies the satisfactory behaviour of the deletion procedure in the presence of AO. Ledolter (1990) has also emphasized the use of error variance to detect the presence of outlier in the time series. Based on this empirical study, he also concluded that the error variance is an appropriate statistic to investigate the presence of outliers of either type.

Schmid (1986, 1990) considered multiple outliers problem, and derived a test of discordancy for AO type of outliers in an AR process. In addition, he investigated the asymptotic behaviour of the test. Schmid (1989) also proposed a UMPU test for AO identification in an AR model. Next, Schmid (1996) introduced an alternative multiple outlier detection and identification test procedure for AR process which is based on observed and predicted values at each time instance. The asymptotic distribution of the test statistic under the hypothesis of outlier free model was also derived. Further, Schmid presented a simulation-based performance comparison of various procedures with the proposed procedure.

Ljung (1993) expressed that analogous to the estimation of outlier parameter in regression model, the estimation of additive outlier in ARMA (p, q) is directly related to estimation of missing or deleted observation and established the relation between leave-k-out diagnostics procedure by Bruce and Martin and likelihood ratio criteria. Ljung, however, pointed out that deletion diagnostic measures are expected to perform well for AO but not IO.

Alternatively, McCulloch and Tsay (1994) and Barnett, Kohn and Sheather (1996, 1997) alternatively used Markov Chain Monte Carlo (MCMC) methods to detect outliers and compute the posterior distribution of the parameters in case of ARIMA.

McCullogh and Tsay (1994) proposed the procedure using Gibbs sampling. They revealed that the Gibbs sampling provides accurate parameter estimation and effective outlier detection for an AR process when the outliers do not occur in patches. However, Justel, Pena and Tsay (2001) showed that the procedure is not efficient in the presence of patches of additive outliers in an autoregressive process. They also pointed out that the leave-k-out procedure cannot efficiently determine the block size of the patches of outliers. Their procedure consists of modification of standard Gibbs sampling and the algorithm presented is shown to work effectively using real life and simulated data.

De Jong and Penzer (1998) did another approach to diagnostic checking of outlying values, level shifts and switches using state space modeling of time series.

Tsay, Pena and Pankratz (2000) stretched out the outlier problem in univariate time series to a vector-valued autoregressive integrated moving average series. They discussed about the effect of multivariate outlier and its impact on the joint and marginal models. The discussion pointed out that the effects depended not only on the outlier size and the model, but also on the interaction between the two. In this study, they proposed and investigated two statistics for various types of outlier detection.

Soe Win (2004) proposed a different diagnostic procedure known as adjustment diagnostics. Two separate models (AR and MA) for two types of outliers (AO and IO) were considered in this procedure. Besides, each observation was treated as a possible outlier of each type in turn and the observed series was appropriately adjusted, taking into account the underlined model and estimation of parameters. The model adjustment was shown to be similar to deletion diagnostics of Ledolter (1990) in case of AO and was also shown to handle the influence of IO on successive observations. Both from theoretical and empirical points of view, the adjustment diagnostics were presented.

## 2. 6 Procedures for Outlier Detection

The ideas to find time series outliers were to examine higher moments, in practice skewness and kurtosis, of the series or to apply some smoothing process. These methods are simple and perhaps useful in some cases, but obviously not sufficient for wide variety of situations encountered in empirical time series analysis. It was therefore necessary to consider more complicated methods and some of these are presented.

In practice, the timing of an intervention event may or may not be known. Often in such cases, when the timing and causes of a series of interventions are known, an appropriate handling can be carried out using intervention analysis that was proposed by Box and Tiao (1965). Following Box and Tiao, time series with unusual observations were analyzed using intervention analysis in the literature (Glass, Willson and Gottman, 1975, Kendall and Ord, 1990, Wei, 1990, Box, Jenkins and Reinsel, 1994, Liu, 2006). In many situations, the timing of intervention is rarely known beforehand and it has significant influence on the analysis of time series. It leads to the general time series outlier analysis. The presence of outliers is often not known at the start of the time series data analysis; additional procedures for detection of outliers and assessment of their possible impacts are important in practice.

There are a number of outlier detection methods. The number of outliers to be detected varies (either one or more than one), and in methods of multiple outliers there is a distinction between tests with known and unknown numbers of outliers to be identified. In time series analysis a well-specified model is a necessary requirement for all methods. The most common models considered are ARIMA and dynamic regression models.

Some of the outlier detection methods are close to one another. Outlier detection methods are also connected to the analysis of missing data. An additive outlier can be treated as if the observation in question were missing, and a new value estimated to replace it. This point is noted and utilized by many authors. It is also interesting to note that many of these methods have got their idea from regression analysis. This is perhaps not so surprising, since regression analysis is the idea of the first developments in outlier detection and modeling, and also of the most advanced and widely accepted methods.

The procedures for outlier detection in time series can be classified into two groups: formal procedure (derived from a general theory) and informal procedure (intuitively reasonable and ad hoc manner). In this chapter, the formal and informal procedures of outlier detection based on available literature are presented.

## 2. 7 Formal Procedure

The well-known formal procedure for detection of outliers in a time series is the likelihood ratio test, which was first proposed by Fox (1972) followed by Chang and Tiao (1983), Tiao (1985), Tsay (1986), Chang, Tiao and Chen (1988) and Chan and Liu (1993). Other references on this topic include Abraham and Box (1979), Martin (1980), Hillmer, Bell, and Tiao (1983).

### 2. 7. 1 Likelihood Ratio Criterion

The idea of using likelihood ratio test to detect outliers in time series was also originally proposed by Fox (1972). He classified time series outliers to type I (additive outlier) and type II (innovational outlier) based on an autoregressive model. The basic idea (in an autoregressive model) is to add a dummy variable for every observation in turn, maximize these likelihoods, and see whether the maximum of the likelihood ratio statistics thus achieved is significant. Fox also suggested the use of more practical simplifications of likelihood ratio test, such as standardized estimated errors in the observations being tested. These were developed by, among others, Muirhead (1986), Chang, Tiao and Chen (1988) and Tsay (1986, 1988), and are also used as a part of a complete outlier modeling strategy.

The models for additive outliers (AO) and innovational outliers (IO) are as described in Equations (2.2.6) and (2.2.7) respectively. These two models can be written in terms of the innovation sequence $a_t$'s as follows:

$$\text{AO:} \quad Y_t = \frac{\theta(B)}{\phi(B)} a_t + \omega P_t^{(T)} \tag{2.7.1}$$

$$\text{IO:} \quad Y_t = \frac{\theta(B)}{\phi(B)} \left( a_t + \omega P_t^{(T)} \right) \tag{2.7.2}$$

Thus, the AO case may be called a gross error model, since only the level of the $t^{th}$ observation is affected. On the other hand, an IO represents an extraordinary shock at time point T influencing $Y_T$, $Y_{T+1}$, . . . , through the dynamic system described by $\dfrac{\theta(B)}{\phi(B)}$.

Let $e_t = \pi(B)\, Y_t$ for $t = 1, 2, \ldots, n$ where $\pi(B) = \dfrac{\phi(B)}{\theta(B)}$. We can write Equations (2.7.1) and (2.7.2), respectively as

$$\text{AO:} \quad e_t = \omega \pi(B) P_t^{(T)} + a_t \tag{2.7.3}$$

$$\text{IO:} \quad e_t = \omega P_t^{(T)} + a_t \tag{2.7.4}$$

In other words, the information about an IO is contained in the residual $e_T$ at that particular point T, whereas that of an AO is scattered over a string of residuals $e_T$, $e_{T+1}$, . . . .

For n available observations, the AO model (2.7.3) can be written as

$$\begin{bmatrix} e_1 \\ \cdot \\ \cdot \\ \cdot \\ e_{T-1} \\ e_T \\ e_{T+1} \\ e_{T+2} \\ \cdot \\ \cdot \\ \cdot \\ e_n \end{bmatrix} = \omega \begin{bmatrix} 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 1 \\ -\pi_1 \\ -\pi_2 \\ \cdot \\ \cdot \\ \cdot \\ -\pi_{n-T} \end{bmatrix} + \begin{bmatrix} a_1 \\ \cdot \\ \cdot \\ \cdot \\ a_{T-1} \\ a_T \\ a_{T+1} \\ a_{T+2} \\ \cdot \\ \cdot \\ \cdot \\ a_n \end{bmatrix} \tag{2.7.5}$$

Let $\hat{\omega}_A$ be the least square estimator of $\omega$ for the AO model. Because $\{a_t\}$ is white noise, from the least squares theory, we have

$$\text{AO:} \quad \hat{\omega}_A = \frac{e_T - \sum_{j=1}^{n-T} \pi_j e_{T+j}}{\sum_{j=0}^{n-T} \pi_j^2}$$

$$= \rho^2 \pi(F)\, e_T \tag{2.7.6}$$

where $\rho^2 = \left(\sum_{j=0}^{n-T} \pi_j^2\right)^{-1}$ and $\pi(F) = (1 - \pi_1 F - \pi_2 F^2 - \ldots - \pi_{n-T} F^{n-T})$, F is the forward shift operator such that $Fe_t = e_{t+1}$. The variance of the estimator is

$$\text{var}(\hat{\omega}_A) = \rho^2 \sigma_a^2 \tag{2.7.7}$$

Similarly, letting $\hat{\omega}_I$ be least squares estimator of $\omega$ for the IO model, we have

$$\text{IO:} \quad \hat{\omega}_I = e_T \tag{2.7.8}$$

and

$$\text{var}(\hat{\omega}_I) = \sigma_a^2 . \tag{2.7.9}$$

Thus, the best estimate of the effect of an IO at time T is the residual $e_T$, whereas the best estimate of the effect of an AO is a linear combination of $e_T, e_{T+1}, \ldots$ and $e_n$ with weight depending on the structure of the time series process. Since $\rho^2 \leq 1$, it is easily seen that var($\hat{\omega}_A$) $\leq$ var($\hat{\omega}_I$)= $\sigma_a^2$ and in some cases, var($\hat{\omega}_A$) can be much smaller than $\sigma_a^2$.

Let $H_0$ denote the null hypothesis that $\omega = 0$ in Equations (2.7.1) and (2.7.2), $H_1$ denote the situation $\omega \neq 0$ in Equation (2.7.1) for AO and $H_2$ denote the situation $\omega \neq 0$ in Equation (2.7.2) for IO. The likelihood ratio test statistics for AO and IO are

$$H_0 \text{ vs } H_1 : \quad \lambda_{1,T} = \frac{\hat{\omega}_A}{\rho \sigma_a}$$

$$H_0 \text{ vs } H_2 : \quad \lambda_{2,T} = \frac{\hat{\omega}_I}{\sigma_a} \tag{2.7.10}$$

Under the null hypothesis $H_0$, the statistics $\lambda_{1,T}$ and $\lambda_{2,T}$ both have the standard normal distribution.

The likelihood ratio method further leads to the criteria

$$\text{AO} \quad : \quad \max_{t=1,\ldots,n} |\lambda_{1,T}|$$

$$\text{IO} \quad : \quad \max_{t=1,\ldots,n} |\lambda_{2,T}|$$

for testing the possibility of an AO or IO, respectively, at an unknown position in the series $Y_1, \ldots, Y_n$.

A simple rule was mentioned by Fox (1972) as a possible way to distinguish between AO and IO. At any suspected point T, the possible outlier is classified as an AO if $|\lambda_{1,T}| > |\lambda_{2,T}|$ and it is classified as an IO if $|\lambda_{1,T}| \leq |\lambda_{2,T}|$.

In practice the ARMA parameters and $\sigma_a^2$ are usually unknown. Estimates of these parameters, together with that of $\omega$ under either the AO or the IO case, can be obtained by maximizing the likelihood function of $(\phi_1, \ldots, \phi_p, \theta_1, \ldots, \theta_q, \omega, \sigma_a^2)$ in the same fashion as that described by Box and Jenkins (1976). Based on these estimates, the likelihood ratios can be computed accordingly for testing the hypotheses, one against another, in Equation (2.7.10).

**An Iterative Procedure for Outlier Detection**

The procedure for the detection of outliers in a time series at unknown positions is as follows:

**Step 1**

Model the series $\{Y_t\}$ by assuming that there is no outlier. From the estimated model, compute the residuals, that is,

$$\hat{e}_t = \hat{\pi}(B)Y_t = \frac{\hat{\phi}(B)}{\hat{\theta}(B)}Y_t$$

where $\hat{\phi}(B) = \left(1 - \hat{\phi}_1 B - \ldots - \hat{\phi}_p B^p\right)$ and $\hat{\theta}(B) = \left(1 - \hat{\theta}_1 B - \ldots - \hat{\theta}_q B^q\right)$. Let

$$\hat{\sigma}_a^2 = \frac{1}{n}\sum_{t=1}^{n} \hat{e}_t^2$$

be the initial estimate of $\sigma_a^2$.

**Step 2**

Calculate $\hat{\lambda}_{1,t}$ and $\hat{\lambda}_{2,t}$ for $t = 1, 2, \ldots, n$, using the estimated model. Define

$$\hat{\lambda}_T = \max\left\{\left|\hat{\lambda}_{1,t}\right|, \left|\hat{\lambda}_{2,t}\right|\right\}$$

for $t = 1, 2, \ldots, n$, where T denotes the time when the maximum occurs. If $\hat{\lambda}_T = \left|\hat{\lambda}_{1,T}\right| > C$, where C is a predetermined positive constant, then there is the possibility of an AO at time T with its effect estimated by $\hat{\omega}_A$ in Equation (2.7.6). The effect of AO can be removed by defining new residuals

$$\tilde{e}_t = \hat{e}_t - \hat{\omega}_A \hat{\pi}(B) P_t^{(T)}$$

for $t \geq T$.

If $\hat{\lambda}_T = \left|\hat{\lambda}_{2,T}\right| > C$, then there is the possibility of an IO at time T. The impact of IO is estimated by $\hat{\omega}_I$ in Equation (2.7.8). Then, the effect can be eliminated by defining a new residual

$$\tilde{e}_T = \hat{e}_T - \hat{\omega}_I = 0$$

at time T.

In practice, Chang et. al. (1988) recommended using $C = 3$ for high sensitivity, $C = 3.5$ for median sensitivity and $C = 4$ for low sensitivity in the outlier detecting procedure when the length of the series is less than 200.

In either of preceding cases, a new estimate $\tilde{\sigma}_a^2$ is computed from modified residuals.

**Step 3**

If an IO or an AO is identified in Step 2, recompute $\hat{\lambda}_{1,T}$ and $\hat{\lambda}_{2,T}$ based on the same initial estimates of time series parameters, but using the modified residuals $\tilde{e}_t$'s and the estimate $\tilde{\sigma}_a^2$, and repeat Step 2.

**Step 4**

Continue to repeat Steps 2 and 3 until no further outliers can be identified.

**Step 5**

Suppose that Step 4 terminated and k outliers have been tentatively identified at times $T_1, T_2, \ldots$, and $T_k$. Treat these time points as known and estimate the outlier parameters simultaneously using general outlier model of the form

$$Y_t = \sum_{j=1}^{k} \omega_j V_j(B) P_t^{(T_j)} + \frac{\theta(B)}{\phi(B)} a_t \tag{2.7.11}$$

which is equivalent to Equation (2.2.10) where $V_j(B) = 1$ for an AO and $V_j(B) = \frac{\theta(B)}{\phi(B)}$ for an IO at time $T_j$.

Treating Equation (2.7.11) as the suggested model, we start the outlier detection stage again. If no other outliers are found, we stop. Otherwise, the estimation stage is repeated, with the newly identified outliers incorporated into the model (2.7.11), until no more outliers can be found and all of the outlier effects have been simultaneously estimated with the time series parameters. Thus, we have the following fitted outlier model:

$$Y_t = \sum_{j=1}^{k} \hat{\omega}_j V_j(B) P_t^{(T_j)} + \frac{\hat{\theta}(B)}{\hat{\phi}(B)} a_t \qquad (2.7.12)$$

where $\hat{\omega}_j$, $\hat{\phi}(B) = \left(1 - \hat{\phi}(B) - \ldots - \hat{\phi}_p B^p\right)$ and $\hat{\theta}(B) = \left(1 - \hat{\theta}_1 B - \ldots - \hat{\theta}_q B^q\right)$ are obtained in the final iteration.

## 2. 8 Informal Procedure

The informal procedures of outlier detection in time series which are available in literature include, Influence Function Method, Q Statistics, Leave-k-out Diagnostics and Likelihood Displacement. Chernick, Downing and Pike (1982) suggested the method of detecting outliers by examining the influence function matrix of the estimated autocorrelations. Abraham and Chuang (1989) considered the Q Statistics used in regression analysis for detection of outliers in time series. Bruce and Martin (1989) also proposed the leave k-out diagnostics as an informal procedure. Ledolter (1990) also considered the likelihood displacement measure for the model perturbation in the case of time series observation. Soe Win (2004) proposed adjustment diagnostic measures based on error variance (ADV) and developed Statistical Time Series Diagnostic Software (STDS) for the detection of outliers in time series.

### 2. 8 .1 Influence Function Method

Influence is a concept that has direct applications to the study of outliers. Influential observations are also potential outliers, so detecting them is a natural complement to the detection of outliers. It should be noted that not all outliers are influential, and similarly, not all influential observations are outliers. Furthermore, influence measures can perhaps help outlier detection with the problems caused by masking and smearing.

Chernick, Downing and Pike (1982) investigated the effect of outliers on time series data by considering the influence function for the autocorrelation of a stationary time series. They defined an influence function matrix for the autocorrelations and showed how it can be used as a graphical tool for detecting outliers. They also accept been simultaneously estimated with the time series parameters. Thus, we have the following fitted outlier model:

$$Y_t = \sum_{j=1}^{k} \hat{\omega}_j V_j(B) P_t^{(T_j)} + \frac{\hat{\theta}(B)}{\hat{\phi}(B)} a_t \qquad (2.7.12)$$

where $\hat{\omega}_j$, $\hat{\phi}(B) = \left(1 - \hat{\phi}(B) - \ldots - \hat{\phi}_p B^p\right)$ and $\hat{\theta}(B) = \left(1 - \hat{\theta}_1 B - \ldots - \hat{\theta}_q B^q\right)$ are obtained in the final iteration.

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Chernick, Downing and Pike (1982) investigated the effect of outliers on time series data by considering the influence function for the autocorrelation of a stationary time series. They defined an influence function matrix for the autocorrelations and showed how it can be used as a graphical tool for detecting outliers. They also accept that the influence function for bivariate correlation can be a useful tool for detecting outliers.

The popular Box and Jenkins approach of time series model building includes four stages: model identification, parameter estimation, diagnostic checking and forecasting. The model identification part relies heavily on the behavior of the sample autocorrelation function. Since individual outliers can have dramatic effects on several correlations, this may have implications on the identification phase of Box and Jenkins method when errors are suspected in the series. In such cases, the influence function matrix for the data set may be computed before starting the Box and Jenkins method.

An influence function for an estimate is the result of an infinitesimal change in the weight given to an observation in the theoretical distribution function. The influence function for an estimate depends on the parameters being estimated; the observation vector whose influence is being measured and the distribution function of that observed vector. The parameter can be considered as a functional of the distribution function F and is commonly written T(F). Often the estimator under consideration can be expressed as $T(F_m)$, where $F_m$ is the empiric distribution function that is, $F_m(y)$ is the fraction of the m sample vectors x whose coordinates are all less than or equal to the coordinates of y. The influence function defined by Hampel (1974) is given by

$$I(F, T(F), x) = \lim_{\varepsilon \to 0} \{T(1 - \varepsilon) F + \varepsilon\, \delta\, x) - T(F)\} / \varepsilon \qquad (2.8.1)$$

when the limit on the right side exists. In Equation (2.8.1), x is the point of interest in the observation space, $\varepsilon$ is a positive real number, and $\delta x$ is the distribution function that has all its probability mass concentrated at the point x. An influence function therefore measures the effect of a small change or perturbation of the data distribution on the value of a statistic or an estimate of interest.

Let $\rho_k$ denote the autocorrelation at lag k for a stationary time series $X_t$, $t = 1, 2, \ldots, \infty$. Let $\mu = E(X_t)$ and $\sigma^2 = V(X_t)$. It is convenient to let $Y_t = (X_t - \mu) / \sigma$ for each t. This transformation from $X_t$ to $Y_t$ does not affect $\rho_k$ and therefore $I(F, \rho_k, x) = I(H, \rho_k, y)$, where H is the distribution for $(Y_t, Y_{t+k})$. Then

$$I(H, \rho_k, (Z_t, Z_{t+k})) = Z_t Z_{t+k} - \rho_k (Z_t^2 + Z_{t+k}^2)/2 \qquad (2.8.2)$$

where H is a bivariate distribution function with mean zero, unit variance and covariance $\rho_k$.

The influence of any pair of observations k units apart on the estimate of $\rho_k$ can be computed based on the Equation (2.8.2). When $\rho_k$, $\sigma$ and $\mu$ are not known, estimates can be used to calculate the influence function.

An n x m matrix, where n is the number of observations and m is a fixed number of lags (m < n) has its (j, k) entry given by $I(H, \rho_k, (y_j, y_{j+k}))$ where $y_j$ is the $j^{th}$ standardized observation. The observation $y_t$ influences several lagged autocorrelation estimates. It appears in the computation of every element in the $t^{th}$ row and also in the diagonal elements of the preceding rows beginning in column 1 of row t-1 and proceeding up and to the right. An outlier will often have a very large positive or negative influence on each estimate of correlation. Hence, if all the elements in the $t^{th}$ row and the above diagonal are large in absolute value, this will indicate that the $t^{th}$ observation is probably an outlier. This "clothes-pin" pattern in the matrix directs one's attention to the suspect observation.

Define

$$U_{i,k,1} = \left\{ \frac{(y_i + y_{i+k})}{\sqrt{1+\rho_k}} + \frac{(y_i - y_{i+k})}{\sqrt{1-\rho_k}} \right\} / 2 \qquad (2.8.3)$$

and

$$U_{i,k,2} = \left\{ \frac{(y_i + y_{i+k})}{\sqrt{1+\rho_k}} - \frac{(y_i - y_{i+k})}{\sqrt{1-\rho_k}} \right\} / 2 \qquad (2.8.4)$$

It is easy to see that

$$\left(1-\rho_k^2\right) U_{i,k,1} \; U_{i,k,2} = y_i y_{i+k} - \rho_k (y_i^2 + y_{i+k}^2)/2 \qquad (2.8.5)$$

and so

$$I(H, \rho_k, (y_i y_{i+k})) = \left(1-\rho_k^2\right) U_{i,k,1} U_{i,k,2} \qquad (2.8.6)$$

For a stationary Gaussian process with $\mu$, $\sigma$ and $\rho_k$ all known, $U_{i,k,1}$ and $U_{i,k,2}$ are observations from independent standard normal distributions. The quantity $I(H, \rho_k, (y_i, y_{i+k}))$ then has the distribution of a constant times a product of standard normal random variables. This distribution can be used to determine what values for the influence function would be unusually large for a realization from a stationary Gaussian process.

To clearly see the patterns in the influence function matrix, Chernick et. al (1982) proposed choosing a critical value of 1.0 based on the product standard normal distribution. Influence function estimates exceeding this critical value in absolute value are designated + or – depending on the sign of the estimate. Other observations are left blank. The matrix will then appear with patterns of + s and – s and the clothes-pin effect should be evident to the eye. While Chernick et al. did not expect that the stationary and Gaussian assumptions hold in all applications, they believed that this approach provides a method for revealing outliers in a wide variety of situations.

### 2. 8. 2 Q Statistics

Abraham and Chuang (1989) considered some statistics used in regression analysis for detection of outliers in time series. This method was proposed for distinguishing an AO from an IO. A four-step procedure for modeling time series in the presence of outliers was also proposed.

**Some Diagnostic Checking Measures**

Let $Z_t$ be a stationary AR (p) process given by

$$\phi(B)\ Z_t = a_t \tag{2.8.7}$$

where $\phi(B)$ is defined as before . Given a set of observations $z_1, z_2, \ldots, z_n$. We can write

$$\mathbf{Z} = \mathbf{X}\boldsymbol{\phi} + \mathbf{a} \tag{2.8.8}$$

where $\mathbf{Z}' = (z_{p+1}, \ldots, z_n)$, $\boldsymbol{\phi}' = (\phi_1, \ldots, \phi_p)$, $\mathbf{a}' = (a_{p+1}, \ldots, a_n)$ and

$$\mathbf{X} = \begin{bmatrix} z_p & z_{p-1} & \cdots & z_1 \\ z_{p+1} & z_p & \cdots & z_2 \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & & \cdot \\ z_{n-1} & z_{n-2} & \cdots & z_{n-p} \end{bmatrix}$$

Then the conditional least squares (CLS) estimate of $\boldsymbol{\phi}$ is given by

$$\hat{\varphi} = (\mathbf{X}'\mathbf{X})^{-1}\ \mathbf{X}'\mathbf{Z}, \tag{2.8.9}$$

the fitted values are

$$\hat{\mathbf{Z}} = \mathbf{X}\hat{\varphi} = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Z} = \mathbf{HZ} \tag{2.8.10}$$

where $\mathbf{H} = \mathbf{X}(\mathbf{X'X})^{-1}\mathbf{X'}$ and the residuals are $\mathbf{e} = (\mathbf{I} - \mathbf{H})\mathbf{Z}$ . In a linear regression model, **X** is assumed to be a constant matrix and the $Z_t$'s are assumed independent. An observation could be deleted without affecting the consecutive ones and the deletion of an equation in (2.8.8) is equivalent to the deletion of an observation. However, in time series context, a suspected observation $Z_t$ is involved not only in one equation but in p+1 consecutive equations of (2.8.8). Thus, it may be necessary to delete not only one equation but p+1 equations from (2.8.8).

Suppose that there is one suspected observation at t = T. The matrix **X**, vectors **Z** and **e** can be partitioned as follows:

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \\ \mathbf{X}_3 \end{bmatrix} \begin{matrix} (T-p)\times p \\ k\times p \\ (n-T-k)\times p \end{matrix} \qquad \mathbf{Z} = \begin{bmatrix} \mathbf{Z}_1 \\ \mathbf{Z}_2 \\ \mathbf{Z}_3 \end{bmatrix} \begin{matrix} (T-p)\times 1 \\ k\times 1 \\ (n-T-k)\times 1 \end{matrix} \qquad \mathbf{e} = \begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \\ \mathbf{e}_3 \end{bmatrix} \begin{matrix} (T-p)\times 1 \\ k\times 1 \\ (n-T-k)\times 1 \end{matrix}$$

where k is the number of equation that are to be deleted. The residuals **e** can be expressed in the partitioned form as

$$\mathbf{e} = \begin{bmatrix} \mathbf{I}-\mathbf{H}_{11} & -\mathbf{H}_{12} & -\mathbf{H}_{13} \\ -\mathbf{H}_{21} & \mathbf{I}-\mathbf{H}_{22} & -\mathbf{H}_{23} \\ -\mathbf{H}_{31} & -\mathbf{H}_{32} & \mathbf{I}-\mathbf{H}_{33} \end{bmatrix} \begin{bmatrix} \mathbf{Z}_1 \\ \mathbf{Z}_2 \\ \mathbf{Z}_3 \end{bmatrix} \tag{2.8.11}$$

where

$$\mathbf{H}_{ij} = \mathbf{X}_i\,(\mathbf{X'X})\mathbf{X'}_j \qquad , i, j = 1, 2, 3 \tag{2.8.12}$$

Consider the statistics

$$\mathbf{Q}_{k(t)} = \mathbf{e'}_2\,(\mathbf{I} - \mathbf{H}_{22})^{-1}\,\mathbf{e}_2 \tag{2.8.13}$$

and

$$\mathrm{AP}_{k(t)} = (1 - \mathrm{Q}_{k(t)} / \mathrm{RSS})\,|\mathbf{I} - \mathbf{H}_{22}| \tag{2.8.14}$$

where RSS is the residual sum of squares. When $k = 1$, $\mathbf{e'}_2 = e_T$ and when $k = p+1$, $\mathbf{e'}_2 = (e_T, \ldots, e_{T-p})$. Now $Q_{k(t)}$ could be decomposed into two terms:

$$\begin{aligned} Q_{k(t)} &= \mathbf{e'}_2\mathbf{e}_2 + (\hat{\varphi} - \hat{\varphi}^*)'\,(\mathbf{X'}_1\mathbf{X}_1 + \mathbf{X'}_3\mathbf{X}_3)(\hat{\varphi} - \hat{\varphi}^*) \\ &= Q_{k1(t)} + Q_{k2(t)} \end{aligned} \tag{2.8.15}$$

where $\hat{\varphi}^* = (\mathbf{X'}_1\mathbf{X}_1 + \mathbf{X'}_3\mathbf{X}_3)^{-1}(\mathbf{X'}_1\mathbf{Z}_1 + \mathbf{X'}_3\mathbf{Z}_3)$ is the estimate of $\varphi$ after deleting k equations from (2.8.8). Note here that $Q_{k2(t)}$ and Cook's statistic with k observations deleted

$$C_{k(t)} = e'_2 (I - H_{22})^{-1} H_{22} (I - H_{22})^{-1} e_2 / (p\hat{\sigma}^2) \qquad (2.8.16)$$

are related as follows:

$$\frac{1}{1-\lambda_1} p\hat{\sigma}^2 \frac{C_{k(t)}}{Q_{k2(t)}} \leq \frac{1}{1-\lambda_k} \qquad (2.8.17)$$

where $\hat{\sigma}^2 = RSS / (n - p - 1)$ and $\lambda_1$ and $\lambda_k$ are the smallest and the largest eigenvalues of $H_{22}$. The statistics $Q_k$, $Q_{k1}$ and $Q_{k2}$ are useful indicators for outliers detection.

**Patterns of the Q Statistics**

The statistics defined in Equations (2.8.13) and (2.8.15) are functions of $e'_t$ s and $h_{ij}$ $[i,j = t, \ldots, (t+k-1)]$. Their behaviour is different for AO and IO outliers. They may be used not only to detect an outlier but also distinguish an AO from an IO.

A suspected AO outlier at $t = T$ will affect $Z_T$ by $\delta$ and consequently, $e_{T+1}$ by $\phi_i \delta$ $(i = 0, 1, \ldots, p;\ \phi_0 = 1)$. An IO outlier with affect $e_T$ by $\delta$ and hence $Z_{T+1}$ by $\psi_i \delta$ $(i = 0, 1, \ldots)$, where $\psi_i$ is the coefficient of $B^i$ in $\psi(B) = \phi^{-1}(B) = 1 - \psi_1 B - \psi_2 B^2 \ldots (\psi_0 = 0)$.

For illustration of the patterns, consider an AR(1) process. Suppose that k=1; then

$$H_{22} = h_{TT} = Z_{T-1}^2 / \sum_{t=2}^{n} Z_{t-1}^2 ,$$

$$Q_{1(T)} = e_T^2 (1 - Z_{T-1}^2 / \sum_{t=2}^{n} Z_{t-1}^2)^{-1} = e_T^2 (1 - h_{TT})^{-1},$$

$$Q_{11(T)} = e_T^2 \text{ and}$$

$$Q_{12(T)} = e_T^2 Z_{T-1}^2 / \sum_{t \neq T} Z_{t-1}^2 = e_T^2 [h_{TT} / (1 - h_{TT})] .$$

$Q_{11}$ depends only on $e_T$, whereas $Q_1$ and $Q_{12}$ depend on $e_T$ and $h_{TT}$; however, $h_{TT}$ is relatively small compared with 1 and the behavior of $Q_1$ is dominated by $e_T$. Now $h_{TT} / (1 - h_{TT})$ is a monotone function of $h_{TT}$, thus the behavior of $Q_{12}$ depends on $e_T^2 Z_{T-1}^2$.

If the outlier at $t = T$ is an AO, then $e_T$ and $e_{T+1}$ are affected, and hence $Q_{11(T)}, Q_{11(T+1)}$ and $Q_{1(T)}$, $Q_{1(T+1)}$ are large compared with the rest. Although $Q_{12(T)}$ and $Q_{12(T+1)}$ are influenced by the outlier at $t = T$, then the latter is the larger because $e_{T+1}$ and $Z_T$ are affected by the outlier.

If the outlier is IO, then only $e_T$ is affected, which implies that $Q_{1(T)}$ and $Q_{11(T)}$ are large compared with others. The behavior of $Q_{12(T)}$ is less reliable, since observations $z_T, \ldots, z_n$ are all affected.

The patterns of these statistics for higher order (p > 1) processes are similar and are summarized in Table (2.1). In general, it is indicated that $Q_k$ ( or $Q_{k1}$ ) is more useful in detecting outliers than $Q_{k2}$.

**Table (2.1)**

**Patterns of Q statistics Assuming an Outlier at t = T**

| Statistics | IO outlier | AO outlier |
|---|---|---|
| $Q_{11}$, $Q_1$ deleting one equation (k=1) | Large value at t =T and small values elsewhere. | The values at t = T, T+1, . . ., T+p are affected. |
| $Q_{12}$, deleting one equation (k=1) | The values at t =T, T+1, . . . are affected (less reliable). | The values at t=T, T+1, . . . , T+p are affected. |
| $Q_{(p+1)1}$,$Q_{(p+1)}$, deletion p+1 equations (k=p+1) | Large values at t =T-p, T-p+1, . . . , T, and small values elsewhere. | The values at t = T-p, T-p+1, . . . , T+p are affected, with the largest value at t =T. |
| $Q_{(p+1)2}$ deleting p+1 equations (k=p+1) | The values at t =T-p, . . ., T, . . are affected (less reliable). | The values at t = T-p, T-p+1, . . . , T+p are affected, with the largest value at t =T. |

**Approximations**

In practice, the identity of the outlier may not be known. Hence the values of $Q_{k(t)}$, $Q_{k1(t)}$ and $Q_{k2(t)}$ have to be computed for all t = p+1, p+2, . . ., (n-k+1) and this requires (n-k-p+1) inversions of the matrix (**I-H**$_{22}$), which may invite numerical problems. Recursive computational procedures to avoid this inversion can be time-consuming and may result in large rounding errors. If off-diagonal elements, $-h_{ij}$ of (**I-H**$_{22}$) are small in absolute value, the following approximation, in which no matrix inversion is performed, can be made:

$$Q_{k(t)} \approx \sum_{i=t}^{t+k-1} e_t^2 /(1-h_{ii}) \qquad (2.8.18)$$

This approximation is usually adequate in large samples. Once $Q_{k(t)}$ is obtained, $Q_{k2(t)}$ can be obtained by subtracting $Q_{kl(t)}=e_2'e_2$ from $Q_{k(t)}$. These approximations are also attractive because the exact forms require the computation and the storage of the symmetric band matrix **H** (that is, $h_{t,t-k}, \ldots, h_{t,t+k}$ for all t ), whereas the approximations require there only for the diagonal elements of **H**. Note that when k=1 (deleting one equation) the exact values and the approximations are the same.

**Model Building Procedure**

Abraham and Chuang (1989) proposed the following model building procedure, based on the Q statistics.

**Step 1**

Use any model selection technique to identify a tentative order (p', q'), which may not be the true order (p, q). Choose a $p^* > p' + q'$.

**Step 2**

Outlier Detection: Estimate $\boldsymbol{\varphi}^*$ by CLS method and compute of $Q_k$ (and/ or $Q_{k2}$ ) for k = 1 and $k = p^*+1$. Determine the outlier and its type based on the plots of $Q_k$ (and / or $Q_{k1}$, $Q_{k2}$ ). The significance tests based on the maximum of these statistics may also be used. Go to Step 4 if there are no outliers; otherwise continue to Step 3.

**Step 3**

Cleaning the series: Let T be the position of the outlier identified in Step 2. If the outlier type is AO, then delete equations $(T-p^*)$ to T from Equation (2.8.10) to obtain the estimate, $\hat{\boldsymbol{\varphi}}^*$. We now adjust the $T^{th}$ observation, using the predictive mean of $Z_T$ conditional on all other observations, $E(Z_T / Z_t, t \neq T)$ ; that is , replace $Z_t$ by

$$\tilde{Z}_t = Z_t \qquad ,t \neq T$$

$$= \sum_{j=1}^{p^*} \tilde{\eta}_j (Z_{t-j} + Z_{t+j}) \qquad ,t = T \qquad (2.8.19)$$

where $\tilde{\eta}_j = (\hat{\phi}_j^* - \sum_{t=1}^{p^*-j} \hat{\phi}_t^* \hat{\phi}_{t-j}^*)/(1 + \sum_{t=1}^{p^*} \hat{\phi}_t^{*2})$, $(j = 1, 2, \ldots, p^*)$ (Abraham and Box, 1979). If the outlier is IO, delete the $T^{th}$ equation from (2.8.10) to estimate $\boldsymbol{\varphi}^*$ and adjust the observations as follows:

$$\begin{aligned}\tilde{Z}_t &= Z_t & ,t<T \\ &= Z_t - \hat{e}_t^* & ,t=T \\ &= Z_t - \hat{\psi}_{t-T}^* \hat{e}_T^* & ,t>T \end{aligned} \qquad (2.8.20)$$

where $\hat{e}_t^*$ is the residual corresponding to the estimate $\hat{\varphi}^*$ and $\hat{\psi}_j^*$ is the coefficient of $B^i$ in $1-\hat{\psi}_1^* B - \hat{\psi}_2^* B^2 - \ldots = (1-\hat{\phi}_1^* B - \ldots - \hat{\phi}_p^* B^{p^*})^{-1}$.

With the cleaned series, we go to Step 2 and repeat Steps 2 to 3 until no further outlier are found.

**Step 4**

Specification: Use the cleaned series in the last iteration to specify a tentative model. This model is then estimated using maximum likelihood and the tentative strategy of model building discussed by Box and Jenkins (1976) is adopted.

### 2. 8. 3 Leave-k-out Diagnostics

Bruce and Martin (1989) proposed "leave-k-out diagnostics" for the effects of outliers in time series. Leave-k-out refers to deleting k consecutive observations from the series, treating them as missing and replacing them with their predicted values, using again an appropriate method for handling missing values in an ARIMA model. The effects of this perturbation for ARIMA model parameters (Diagnostics for Coefficient) and for the error variance (Diagnostics for the Innovation Variance) are then taken as diagnostic tests for the presence of outliers in an estimated model.

**Diagnostics for Coefficient (DC)**

Let $\alpha$ be the r x 1 vector of parameters in ARIMA model and denote the MLE of $\alpha$ by $\hat{\alpha}$. Let A = $\{t_1, t_2, \ldots, t_k\}$ be an arbitrary subset of $\{1, 2, \ldots, n\}$ and let $\hat{\alpha}_A$ denote the MLE with observations $y_{t_1}, y_{t_2}, \ldots, y_{t_k}$ treated as missing. If some of the observations in A have an undue influence on the estimate $\hat{\alpha}_A$, then this will often reveal itself in the form of a substantial difference between $\hat{\alpha}$ and $\hat{\alpha}_A$.

Bruce and Martin (1989) took their leave-k-out diagnostics for coefficients as

$$DC(A) = n(\hat{\alpha} - \hat{\alpha}_A)' I(\hat{\alpha})(\hat{\alpha} - \hat{\alpha}_A) \qquad (2.8.21)$$

where $I(\hat{\alpha}) = C^{-1}(\hat{\alpha})$ and $C(\hat{\alpha})$ is the covariance matrix of $\hat{\alpha}$.

A rough guide is to judge a subset A of points to be influential if the 'p' value of DC (A) based on the $\chi_r^2$ reference distribution is smaller than 0.5 (Cook and Weisberg; 1982). This guideline is not a significant test and marely serves as a general purpose indication of influence.

**Diagnostics for the Innovation Variance (DV)**

The influence of a subset A can also be measured by evaluating the effect of its removal on the MLE of the innovation variance $\hat{\sigma}^2$. Let $\hat{\sigma}_A^2$ be the MLE of $\sigma^2$ with observations at times $t \in A$ treated as missing. The diagnostic is formed in the same manner as earlier.

Thus, Bruce and Martin (1989) proposed to use leave-k-out diagnostic for innovation variance

$$DV(A) = \frac{n}{2}\left(\frac{\hat{\sigma}^2}{\hat{\sigma}_A^2} - 1\right)^2 \qquad (2.8.22)$$

with the reference distribution being a chi-squared distribution with one degree of freedom. Again, a subset A of observations to be influential if the 'p' value for DV(A) is less than 0.5 using a $\chi_1^2$ distribution.

**Diagnostics for Patches**

For independent observations, computational considerations usually result in deletion of a single observation at a time. However, the time series situation differs from the case of independent observations in at least two important ways:

(i) structure is imposed by time ordering and

(ii) influential observations often come in the form of an outlier patch or other local structural change extending over several observations.

Leave-one-out diagnostics can fail to give clear evidence of influence in the case of patchy disturbances such as outliers. Such behaviour might be regarded as a form of masking since the effect of any single outlier in such a patch can be overwhelmed by the effect of the other outliers. This kind of situation is easily dealt with in time series by leaving out k consecutive observations.

**Superiority of DV over DC**

In the regression case, the error variance is a nuisance prameter and therefore has less intuitive appeal as the basis for a diagnostic. However, in time series models, the inovations variance plays a fundamental role and it is the parameter of interest as well as DV has properties than DC.

The major difference between leave-k-out coefficients DC for time series (including k=1) and the usual regression coefficients diagnostics for independent data is a smearing of the effect of an isolated outlier of patch of outliers to adjacent points. A given point may be judged influential using the diagnostics for the coefficients because of an outlier at an adjacent point. For example, in the AR (p) case, an isolated outlier can result in significant values for DC at "p" times before and after the occurance of the outlier. Hence, interpretation of leave-k-out diagnostics for DC is not so clear as in the usual regression case. In contrast, diagnostics for the innovations variance display much smaller, and often negligible, smearing effects.

**Overall Strategy**

Bruce and Martin (1989) presented an overall strategy for ARIMA model fitting using leave-k-out diagnostics. The diagnostics are used in a simple recipe to determine the length of a patch of influential points. This strategy is embedded in an iterative deletion procedure, which often overcomes problems caused by masking. Since in some cases the iterative deletion procedure fails, more flexible subset deletion techniques are introduced.

**Iterative Deletion Strategy**

The masking of influential points (e.g. outliers) by other influential points is a problem encountered in all types of diagnostics. Masking caused by a single patch of outliers can be handled adequately by leave-k-out diagnostics. More subtle types of masking occur when moderate outliers occur close to one another. These types of masking can often be effectively uncovered by an iterative deletion process which consists of removing suspected outliers from the data and recomputing the diagnostics.

To deal with problems caused by masking, Bruce and Martin (1989) built on the initial patch determination strategy as follows.

(a) Run leave-k-out diagnostics on the data, for k = 1, 2, . . . , until either the length of the most influential (significant) patch is determined using the guideline of $k = K_{max}$ where $K_{max}$ is determined by the user. In principle, $K_{max}$ is the length of the longest patch of outliers present in the data. For $n < 250$, setting $K_{max} = 5$ will often reveal most if not all problems with the data. The second case can result from two possibilities: either no influential observations were detected or the length of an influential patch is ill determined. The latter case may be due to a patch of length greater than $K_{max}$, or perhaps the patch length simply cannot be determined from the data. In this case, the most influential patch is determined as that corresponding to the most significant diagnostic.

(b) If no influential points are found, then conclude the analysis. If influential points are found, then delete the most influential points as identified in step (a) and go back to step (a). The new leave-k-out coefficients should be scaled according to the MLE computed with the outliers removed in order to gauge additional influence of the remaining points.

**Model Identification and Residual Analysis**

In practice, the model must be determined by some criteria such as the Box-Jenkins identification procedure. However, outliers may cause improper model specification. To handle order selection in the presence of outliers, we can embed the iterative deletion strategy in an iterative procedure. The initial model order is selected, and the iterative deletion strategy is performed on the initial model. After removing all influential points, the model is identified again. If the same model is selected, then the analysis is concluded. Otherwise, iterative deletion is performed again, and the cycle is repeated until the same model is identified in successive rounds. While this procedure is usually adequate, it may fail in some situations where a poor initial model is selected. If the wrong model is identified, then removal of the influential points under that model may lead to selection of the same model.

In the presence of outliers, the usual prediction residuals are often misleading for identifying the influential observations. Instead, Bruce and Martin (1989) recommended the examination of the residuals based on the predictions formed when the observations identified as influential are treated as missing. Since the predictions are not distorted by influential observations, this procedure reveals outliers more clearly.

After selecting the model, a careful analysis of the influential data points should be carried out. One may be able to categorize influential points as isolated or patches of outliers, or perhaps associate them with a level shift or variance change.

Points diagnosed as outliers can be further classified by type (AO versus IO). A formal way of determining whether an outlier identified by DV is AO or IO is by Fox's (1972) test. A less formal way of determining whether an outlier is AO or IO is to examine a lag-1 scatter plot of the residuals. Martin and Zeh (1977) pointed out that IOs tend to fall near the abscissa and ordinate of such a plot, whereas AOs tend to appear away from the abscissa and ordinate, assuming that robust parameter estimates have been used to form the residuals.

### 2. 8. 4 Likelihood Displacement Measures

Cook (1986, 1987) introduced a general measure of model perturbation on parameter estimates using contours of log likelihood function. For any model M with parameter vector $\lambda$, let $L(\lambda)$ be the log likelihood function and $\hat{\lambda}$ is the maximum likelihood estimator of $\lambda$. Suppose we have a perturbation model M ($\omega$) and let $L_\omega(\lambda)$ and $\hat{\lambda}_\omega$ be the log likelihood function of the perturbation model and the associated maximum likelihood estimator respectively. Thus we have two estimators $\hat{\lambda}$ and $\hat{\lambda}_\omega$ corresponding to the basic model and perturbation model respectively. The likelihood displacement or likelihood distance $LD_\omega(\lambda)$ proposed by Cook is

$$LD_\omega(\lambda) = 2[L(\hat{\lambda}) - L(\hat{\lambda}_\omega)] \qquad (2.8.23)$$

which measures the changes in the log likelihood function due to the influence of perturbation on the parameter estimates. The likelihood displacement provides a theoretical foundation for a general measure to assess the influence of perturbation on the estimates of model parameters.

Cook also proposed modification of likelihood displacement in situation where a subset of parameters is of interest. In particular, let $\lambda_1$ be the parameter of interest when $\lambda = (\lambda_1', \lambda_2')'$ and let $\hat{\lambda}_\omega = (\hat{\lambda}_{1\omega}', \hat{\lambda}_{2\omega}')'$. Further, let $\hat{\lambda}_2(\lambda_1)$ be the maximum likelihood estimator of $\lambda_2$ obtained on maximizing $L(\lambda_1, \lambda_2)$ when $\lambda_1$ is fixed. Hence

$$L(\hat{\lambda}_{1\omega}, \hat{\lambda}_2(\hat{\lambda}_{1\omega})) = \max_{\lambda_2} L(\hat{\lambda}_{1\omega}, \lambda_2)$$

and the proposed likelihood displacement for $\lambda_1$ is

$$LD_\omega(\lambda_1) = 2\,[\,L(\hat{\lambda}) - L(\hat{\lambda}_{1\omega}, \hat{\lambda}_2(\hat{\lambda}_{1\omega})] \qquad (2.8.24)$$

The likelihood displacement measure is often used in regression diagnostics (Cook and Weisberg, 1982). In the regression setup, the model considered is $Y_{n\times 1} = X_{n\times k}\,\Theta_{k\times 1} + \varepsilon_{n\times 1}$ and the model perturbation is the deletion of $i^{th}$ observation. Cook and Weisberg also established that in the case of regression model where the parameter $\Theta$ is of interest, the likelihood displacement is a monotone function of Cook's D statistic.

In case of time series observation, the likelihood displacement diagnostics measure was derived by Ledolter (1990) where a stationary and invertible ARMA(p, q) model

$$\phi(B)\,Z_t = \theta(B)a_t$$

was considered. The model perturbation considered by Ledolter is the deletion of $i^{th}$ observation following deletion diagnostics proposed by Peña (1987) and Bruce and Martin (1989). The deleted observation is treated as an unknown parameter and its estimate is substituted in the observed series to estimate the parameters of interest. For given time series coefficients $\beta = (\phi', \theta')'$,the estimate of deleted observation used by Ledolter is

$$-\sum_{j=1} \rho_j^* \,(Z_{t+j} + Z_{t-j})$$

where $\rho_j^*$ is the lag j inverse autocorrelation of the process. This estimate is a weighted sum of adjacent observations, which is the Brubacher and Wilsom (1976) estimator.

When the time series parameter $\beta$ and error variance $\sigma_a^2$ are both of interest, the aim is to obtain the likelihood displacement in Equation (2.8.23), which reduces to

$$LD_i(\beta, \sigma_a^2) = 2\ [L(\hat{\beta}, \hat{\sigma}_e^2) - L(\hat{\beta}_{(i)}, \hat{\sigma}_{e(i)}^2)] \qquad (2.8.25)$$

where $L(\hat{\beta}, \hat{\sigma}_e^2)\ \alpha -\frac{n}{2}\ln(\hat{\sigma}_e^2) - \frac{n}{2}$ and $L(\hat{\beta}_{(i)}, \hat{\sigma}_{e(i)}^2)$ is obtained on substituting the maximum likelihood estimates under the perturbation model $\hat{\beta}_{(i)}$ and $\hat{\sigma}_{e(i)}^2$ in the log likelihood function

$$L(\beta, \sigma_a^2) \propto -\frac{n}{2}\ln\left(\sigma_a^2\right) - \frac{1}{2\sigma_e^2} SS(\beta) \qquad (2.8.26)$$

where $\beta$ is a set of parameters $(\phi',\theta')',\sigma_a^2$ is the error variance and $SS(\beta) = \sum_{t=1}^{n} a_t^2$.

Then, the maximum likelihood estimate $\hat{\beta}$ can be obtained by minimizing the conditional sum of squares function $SS(\beta)$ and the maximum likelihood estimate of error variance is given by

$$\sigma_e^2 = \frac{1}{n} SS(\hat{\beta})$$

where $SS(\hat{\beta}) = \sum_{t=1}^{n} [\hat{\pi}(B)Z_t]^2$ and $\hat{\pi}(B) = \frac{\hat{\phi}(B)}{\hat{\theta}(B)}$.

Thus, Equation (2.8.26) gives

$$L\left(\hat{\beta}_{(i)},\hat{\sigma}_{e(i)}^2\right) \propto -\frac{n}{2}\ln\left(\hat{\sigma}_{e(i)}^2\right) - \frac{nS_{e(i)}^2}{2\hat{\sigma}_{e(i)}^2}$$

where $S_{e(i)}^2 = \frac{1}{n} SS(\hat{\beta}_{(i)})$ is an additional estimate of error variance and $SS(\hat{\beta}_{(i)}) = \sum_{t=1}^{n} [\hat{\pi}_i(B)Z_t]^2$, which is based on the original observed series without any adjustment of any observation, and using $\hat{\beta}_{(i)}$.

Therefore, the measure of interest $LD_i\left(\beta,\sigma_a^2\right)$ in Equation (2.8.25) reduces to

$$LD_i\left(\beta,\sigma_a^2\right) = 2\left[\left\{-\frac{n}{2}\ln\left(\hat{\sigma}_e^2\right) - \frac{n}{2}\right\} - \left\{-\frac{n}{2}\ln\left(\hat{\sigma}_{e(i)}^2\right) - \frac{nS_{e(i)}^2}{2\hat{\sigma}_{e(i)}^2}\right\}\right]$$

$$= -n\ln\left[\frac{\hat{\sigma}_e^2}{\hat{\sigma}_{e(i)}^2}\right] + n\left[\frac{S_{e(i)}^2}{\hat{\sigma}_{e(i)}^2} - 1\right].$$

In order to obtain likelihood displacement for the error variance $\sigma_a^2$ of interest, the modified likelihood displacement introduced in Equation (2.8.24) with $\lambda_1 = \sigma_a^2$ reduces to

$$LD_i\left(\sigma_a^2\right) = 2\ [\ L\left(\hat{\beta},\hat{\sigma}_e^2\right) - L\left(\hat{\beta},\hat{\sigma}_{e(i)}^2\right)\ ]$$

where $L\left(\hat{\beta},\hat{\sigma}_e^2\right) \propto -\frac{n}{2}\ln\left(\hat{\sigma}_e^2\right) - \frac{n}{2}$ and $L\left(\hat{\beta},\hat{\sigma}_{e(i)}^2\right)$ is obtained on substituting the estimates $\hat{\beta}$ and $\hat{\sigma}_{e(i)}^2$ in Equation (2.8.26). The substitution gives

$$L\left(\hat{\beta},\hat{\sigma}_{e(i)}^2\right) \propto -\frac{n}{2}\ln\left(\hat{\sigma}_{e(i)}^2\right) - \frac{n\hat{\sigma}_e^2}{2\hat{\sigma}_{e(i)}^2}.$$

Thus, the measure of interest can be simplified to

$$LD_i\left(\sigma_a^2\right) = 2\ [\ L\left(\hat{\beta},\hat{\sigma}_e^2\right) - L\left(\hat{\beta},\hat{\sigma}_{e(i)}^2\right)\ ]$$

$$= 2\left[\left\{-\frac{n}{2}\ln\left(\hat{\sigma}_e^2\right) - \frac{n}{2}\right\} - \left\{-\frac{n}{2}\ln\left(\hat{\sigma}_{e(i)}^2\right) - \frac{n\hat{\sigma}_e^2}{2\hat{\sigma}_{e(i)}^2}\right\}\right]$$

$$= -n\ln\left[\frac{\hat{\sigma}_e^2}{\hat{\sigma}_{e(i)}^2}\right] + n\left[\frac{\hat{\sigma}_e^2}{\hat{\sigma}_{e(i)}^2} - 1\right] \tag{2.8.27}$$

As expected, the expression for the likelihood displacement is same as that derived by Ledolter (1990), except for the difference in the estimates, which in this case is based on adjusted series. Ledolter showed that this likelihood displacement is equivalent to the deletion diagnostic based on error variance (DV) developed by Bruce and Martin (1989). For $x = \frac{\hat{\sigma}_e^2}{\hat{\sigma}_{e(i)}^2} - 1$, using the identity $\ln(1+x) \approx x - \frac{x^2}{2}$, it was shown that Equation (2.8.27) can be approximately expressed as

$$LD_i\left(\sigma_a^2\right) \approx -n\left[\left\{\frac{\hat{\sigma}_e^2}{\hat{\sigma}_{e(i)}^2} - 1\right\} - \frac{1}{2}\left\{\frac{\hat{\sigma}_e^2}{\hat{\sigma}_{e(i)}^2} - 1\right\}^2\right] + n\left[\frac{\hat{\sigma}_e^2}{\hat{\sigma}_{e(i)}^2} - 1\right]$$

$$\approx \frac{n}{2}\left[\frac{\hat{\sigma}_e^2}{\hat{\sigma}_{e(i)}^2} - 1\right]^2 = DV_i \tag{2.8.28}$$

It is clear from the derivations that the diagnostics based on likelihood displacement will agree with the diagnostics proposed by Letdolter based on deletion. Further, since the series adjustment in the presence of AO is equivalent to deletion diagnostic, the procedures will coincide when the adjustment is carried out for AO type of outliers.

If the parameter $\beta$ is only of interest, the likelihood displacement in this case reduces to

$$LD_i(\beta) = 2\ [\ L\left(\hat{\beta},\hat{\sigma}_e^2\right) - L\left(\hat{\beta}_{(i)},S_{e(i)}^2\right)\ ] \tag{2.8.29}$$

where $L\left(\hat{\beta},\hat{\sigma}_e^2\right)$ is as before and $L\left(\hat{\beta}_{(i)},S_{e(i)}^2\right)$ is given by substituting the estimates $\hat{\beta}_{(i)}$ and $S_{e(i)}^2$ in Equation (2.8.26), which reduces to

$$L\left(\hat{\beta}_{(i)},S_{e(i)}^2\right)\ \alpha\ -\frac{n}{2}\ln\left(S_{e(i)}^2\right) - \frac{n}{2}.$$

Hence, we get

$$LD_i(\beta) = 2\left[\left\{-\frac{n}{2}\ln(\hat{\sigma}_e^2) - \frac{n}{2}\right\} - \left\{-\frac{n}{2}\ln(S_{e(i)}^2) - \frac{n}{2}\right\}\right]$$

$$= -n\ln\left[\frac{\hat{\sigma}_e^2}{S_{e(i)}^2}\right].$$

Since the estimate of error variance is more sensitive than the estimates of the time series parameters in the presence of outliers (Bruce and Martin; 1989, Ledolter; 1989, 1990), we return to the likelihood displacement diagnostics for the error variance $\hat{\sigma}_a^2$, given by Equation (2.8.27). Based on Equations (2.8.27) and (2.8.28) and following Ledolter (1990), Soe Win (2004) considered the quantity $n\left[\frac{\hat{\sigma}^2}{\hat{\sigma}_{e(i)}^2} - 1\right]$ as the diagnostic measure based on error variance for outliers in time series. The displacement is equal to n times the influence on scale $D_v(i)$ proposed by Peña (1987). Soe Win (2004) introduced adjustment diagnostic measure based on error variance for both AO and IO types of outliers.

Let $$ADV_{S,i} = n\left[\frac{\hat{\sigma}_e^2}{\hat{\sigma}_{e(i),S}^2} - 1\right] \qquad (2.8.30)$$

where S is either AO or IO, $\hat{\sigma}_e^2$ is the estimated error variance based on the observed series, and $\hat{\sigma}_{e(i),S}^2$ is the estimated error variance based on adjusted series with adjustment position i, by outlier type AO or IO.

The two measures introduced in Equation (2.8.30) can be used for obtaining diagnostic plots. The possible position of outliers will be denoted by the large values of $ADV_{S,i}$.

Hence, for diagnosing the presence of an AO or an IO type of outlier, respectively, in the observed series at an unknown position, Soe Win (2004) proposed the criteria

$$\Omega_{AO} = \max_i ADV_{AO,i} = \max_i n\left[\frac{\hat{\sigma}_e^2}{\hat{\sigma}_{e(i),AO}^2} - 1\right] \quad \text{for AO}$$

$$= ADV_{AO,T} \quad \text{and}$$

$$\Omega_{IO} = \max_i ADV_{IO,i} = \max_i n\left[\frac{\hat{\sigma}_e^2}{\hat{\sigma}_{e(i),IO}^2} - 1\right] \quad \text{for IO}$$

$$= ADV_{IO,T} \qquad (2.8.31)$$

The position of the outlier is given by the time point T at which the statistic achieves the maximum, if the computed statistic is significantly large.

However, the type of outlier present in a time series is rarely known. The series adjustment at correct adjustment position by correct type is likely to yield the smallest estimate of error variance, which will result in the large value of the derived diagnostic measure. Thus, Soe Win (2004) proposed likelihood displacement based criteria

$$\Omega^* = \max\{\max_i ADV_{AO,i}, \max_i ADV_{IO,i}\}$$
$$= \max(\Omega_{AO}, \Omega_{IO}) \qquad (2.8.32)$$

For a given set of observations, $\Omega^*$ is compared with a predetermined positive constant C.

If $\Omega^* = \Omega_{AO} = ADV_{AO,T} > C$ then an AO type of outlier at time point T is identified.

If $\Omega^* = \Omega_{IO} = ADV_{IO,T} > C$ and IO type of outlier at time point T is identified.

He refered to the proposed procedure based on $\Omega^*$ by Adjustment Diagnostic based on Variance estimate (ADV).

Notice that the procedure based on $\Omega_A$ is same as that proposed by Ledolter (1990). In order to obtain the cut–off point C, he refered to the discussion by Ledolter (1990).

Ledolter (1990) suggested that rather than comparing with an upper percentile of the reference distribution, a warning value can be considered. For instance, for AR (1) of length 100, a warning value of 14.5 seems suitable as a 95% percentile. The warning value can be used to indicate a particular observation which needs to be scrutinized.

Soe Win (2004) adopted the suggestion of Ledolter to use warning line or warning limit based on the ADV plots and suggest plotting ADV plots for both $ADV_{A,i}$ and $ADV_{I,i}$ to get on initial idea about the type and the level of contamination of the given series. He presented the Statistical Time Series Diagnostics Software (STDS) along has the ADV plot as one of the menus.

**Diagnostic for Multiple Outliers**

In practice, the number of outliers which might be present in an observed time series is rarely known and a procedure which detects the presence of multiple outliers is needed. Identification of multiple outliers is a challenging problem, particularly due to the masking and swamping effects (Barnett and Lewis, 1994). In many practical situations, however, most of the jointly influential observations are detected by employing the single case diagnostic procedures (Chatterjee and Hadi, 1988). In time series, most of the existing multiple outliers detection procedures are used to detect single outlier procedure iteratively (Chang and Tiao, 1988: Chang et al. 1988)

The iterative procedure based on ADV for identification of multiple outliers is as follows.

**Step 1**

Compute the maximum likelihood estimates of the model parameters $\phi$, $\theta$ and error variance based on the observed series where it is assumed to be outlier free. Hence, the estimates are

$$\hat{\beta} = (\hat{\phi}', \hat{\theta}')' \text{ and}$$

$$\hat{\sigma}_e^2 = \frac{1}{n}\sum_{t-1}^{n} \hat{e}_t^2 .$$

The $\psi$ weights and $\pi$ weights are recursively computed as,

$$\hat{\psi}_j = \hat{\phi}_1\hat{\psi}_{j-1} - \hat{\phi}_2\hat{\psi}_{j-2} - \ldots - \hat{\phi}_p\hat{\psi}_{j-p} - \hat{\theta}_j \qquad \text{for } j > 0$$

where $\hat{\psi}_0 = 1, \hat{\psi}_j = 0$ for $j < 0$, and $\hat{\theta}_j = 0$ for $j > q$, and

$$\hat{\pi}_j = \hat{\theta}_1\hat{\pi}_{j-1} + \hat{\theta}_2\hat{\pi}_{j-2} + \ldots + \hat{\theta}_q\hat{\pi}_{j-q} + \hat{\phi}_j \qquad \text{for } j > 0$$

where $\hat{\pi}_0 = -1, \hat{\pi}_j = 0$ for $j < 0$, and $\hat{\phi}_j = 0$ for $j > p$ respectively.

**Step 2**

For i = 1, 2, . . . , n in turn, calculate the estimated outlier parameters at 'i',

$$\hat{\omega}_{AO,i} = \frac{\hat{\pi}(F)\hat{e}_i}{\sum_{j=0}^{n-i}\hat{\pi}_j^2},$$

$$\hat{\omega}_{IO,i} = \hat{e}_i,$$

and adjust the series for all t by using the estimated observations given by

$$Y_{t(i),AO} = Y_t - \hat{\omega}_{AO,i}P_t^{(i)}$$

$$Y_{t(i),IO} = Y_t - \hat{\omega}_{IO,i}\hat{\psi}(B)P_t^{(i)}.$$

Compute the adjustment diagnostic measure,

$$ADV_{S,i} = n\left(\frac{\hat{\sigma}_e^2}{\hat{\sigma}_{e(i),S}^2} - 1\right) \qquad \text{for } S = AO \text{ and } IO$$

where $\hat{\sigma}_{e(i),S}^2 = \frac{1}{n}\sum_{t=1}^{n}\hat{e}_{t(i),S}^2$ is obtained based on adjusted series $Y_{t(i),S}$, S is AO or IO, and $\hat{e}_{t(i),S}$ is the residual using the estimated parameters $\hat{\beta}_{(i),S} = (\hat{\phi}_{(i),S}, \hat{\theta}_{(i),S})$ for the adjusted series $Y_{t(i),S}$.

**Step 3**

Define $\Omega^* = \max(ADV_{AO,T}, ADV_{IO,T})$ where $ADV_{AO,T} = \max_i ADV_{AO,i}$ and $ADV_{IO,T} = \max_i ADV_{IO,i}$, T is the time point where the maximum occurs. If $\Omega^* < C$, go to Step 4.

If $\Omega^* = ADV_{AO,T} > C$, where C is the predefined positive value, then there is an AO at time T with its effect $\hat{\omega}_{AO,T}$. Take the adjusted series with observation $Y_t$ replaced by $Y_{t(T),AO}$ for all t, and go to Step 1.

If $\Omega^* = ADV_{IO,T} > C$, then there is an IO at time T with its effect $\hat{\omega}_{IO,T}$. Take the adjusted series with observation $Y_t$ replaced by $Y_{t(T),IO}$ for all t, and go to Step 1.

**Step 4**

Suppose 'm' number of outliers are identified with the positions $T_j$, for $j = 1, 2, \ldots, m$. The model is modified to

$$Y_t = \sum_{j=1}^{m}\omega_j V_j(B) P_t^{(T_j)} + \frac{\theta(B)}{\phi(B)} a_t \qquad (2.8.33)$$

where $V_j(B) = 1$ for AO type and $V_j(B) = \frac{\theta(B)}{\phi(B)}$ for IO type at $t = T_j$. The simultaneous estimation is carried out to get the final estimates for a set of parameters $\beta = (\omega', \phi', \theta')'$ where $\omega' = (\omega_1, \omega_2, \ldots, \omega_m)$ and error variance $\sigma_a^2$ using the maximum likelihood estimation procedure.

The multiple outliers in time series can occur in isolation or in patch which are called as isolated outliers or patch outliers respectively (Marin, 1979; Bruce and Martin, 1989). This procedure can work satisfactorily for isolated outliers.

## 2.9 Advantages and Disadvantages of Outliers Detection Methods

The idea of using likelihood ratio test to detect outliers in time series was originally proposed by Fox (1972). It was later developed by Muirhead (1986), Chang, Tiao and Chen (1988) and Tsay (1986 and 1988). It has become almost a standard method for detecting outliers in time series. It is already featured in some computer software packages (for example, SPSS) and easy to understand as well as seem to work reasonably well in most practical situations, especially when used iteratively (Tsay, 1988 and Chen and Liu, 1993). Furthermore, this method can classify the types of outliers.

Chernick, Downing and Pike (1982) presented an alternative method for outlier detection based on influence functions for the autocorrelation function in time series data. The influence measures can perhaps help outlier detection with the problem caused by masking and smearing but this method cannot separate the type of the detected outliers in a time series.

Q Statistics for detection of outlier was also proposed by Abraham and Chuang (1989) and they investigated the effect of deletion of k observations on Q Statistics. These Q Statistics are used to detect outliers in linear model data and they are transferred to the time series context. This is an alternative procedure which can identify and distinguish an AO outlier from an IO outlier. The pattern of Q Statistics is valid even when the moels are overfitted. The strategy is simple and intuitively appealing and it seems to work reasonably well in a number of examples. It can easily be made a routine part of existing time series software. But the computation of Q Statistics require a number of inversions of the matrix, which may result numerical problems, can be time consuming and may invite rounding errors. Therefore, the approximations in computing Q Statistics are used and these approximations are usually adequate in large sample, and are also attractive because approximations do not require a lot of computations and storage of the matrices.

A leave-k-out diagnostics approach to detect outliers in time series was proposed by Bruce and Martin (1989). Leave-k-out refers to deleting k consecutive observations from the series, treating them as missing and replacing them with their predicted values, using again an appropriate method for handling missing values in an ARIMA model. It seems that these diagnostics are the most effective with increasing numbers of deleted and treated as missing. In this way, they can be effectively used to determine the patch length, that is, the number of consecutive observations considered as outliers. This may be a good way to overcome some of the problems caused by masking and smearing. Some problems remain particularly when a single outlier and a patch of outlier lie close to each other. These can possibly be solved by deleting the observations first identified as outliers and then calculating a new set of diagnostics for the remaining observations, and so on, until no outliers are found. Another minor problem is in presenting the results of leave-k-out diagnostics. Bruce and Martin drew bar plots of their statistics for the whole series for increasing values of k and this may result in a large number of graphs. Moreover, this method could not separate the type of identified outliers.

Cook (1986, 1987) introduced a general measure of model perturbation on parameter estimates using log likelihood function. He also proposed modification of likelihood displacement in situation where a subset of parameters is of interest. This likelihood displacement measure is often used in regression diagnostic (Cook and Weisberg, 1982). The likelihood displacement diagnostics measure was derived by Ledolter (1990) in the case of time series observations. The model perturbation considered by Ledolter is the deletion of observation following deletion diagnostics proposed by Bruce and Martin (1989). The method of deletion diagnostics works well in the presence of AO but the IO poses a problem because of its dynamic nature (Chen and Liu, 1993 and Ljung, 1993).

Soe Win (2004) proposed adjustment diagnostics measure based on error variance (ADV) to identify the correct type of outlier and developed Statistical Time Series Diagnostic Software (STDS) to diagnose the outliers in time series. The ADV procedure can work well for isolated outliers but does not result satisfactorily in handling patch outliers.

# CHAPTER III

# EFFECTS OF OUTLIERS IN MODEL IDENTIFICATION AND PARAMETER ESTIMATION

## 3.1 Outliers Generated by Simulation

In time series analysis, the presence of an outlier and its position in the observed time series produces the unpleasant consequences and it becomes an important issue for the analysis of time series. Tolvi (1998) pointed out that the presence of outliers affects the autocorrelation structure of a time series and therefore they also bias the estimated autocorrelation function (ACF), and partial autocorrelation function (PACF). These biases can be severe and they depend on, besides the obvious attributes like the number, type, magnitude and position of the outliers, also the underlying model and its autocorrelation structure. ARMA model identification is traditionally based on the estimated autocorrelations and partial autocorrelations, and will in the presence of outliers therefore be misleading, unless outliers are somehow taken into account. Also, the effect of outliers causes substantial biases in estimated parameters and it can be either major or minor effects based on the types of outliers. Outliers have also effects on the residuals as well as error variance. This effect has several consequences for any further analysis of the residuals.

In this chapter, the effects of two types of outliers on the analysis of AR(1), MA(1) and ARMA(1,1) series using simulated data are separately studied. The values of outlier parameters considered are $\omega = 0, 1, \ldots, 10$ to highlight the effects of AO and IO on the mean and variance, parameter estimates, error variances, as well as some important statistics for the model identification such as autocorrelation function (ACF), and partial autocorrelation function (PACF), Akaike's Information Criterion (AIC) and Bayesian Information Criterion (BIC).

In order to study the effect of outliers on various estimators, each of 1000 outlier free series of AR(1) with $\phi = 0.5$, MA(1) with $\theta = -0.3$ and ARMA(1, 1) series with $\phi = 0.5$ and $\theta = -0.3$ for $n = 100$, $\mu = 0$ and $\sigma_a^2 = 1$ were generated using S-PLUS Software. An AO and an IO with outlier parameter $\omega = 0, 1, \ldots, 10$ was introduced at $t = 50$ for each generated series.

For each of the generated series, the mean and variance of each series, the parameters of interest, namely the time series parameter $\phi$ for AR(1), $\theta$ for MA(1), both $\phi$ and $\theta$ for ARMA(1,1), the error variance $\sigma_a^2$, the lag-1 autocorrelation and partial autocorrelation functions, AIC and BIC values were estimated.

## 3.2 Effects on Mean and Variance

The observed series $Y_t$ with AR(1) model and a single AO type outlier at T is given by

$$Y_t = \omega P_t^{(T)} + Z_t \tag{3.2.1}$$

where $Z_t = \phi Z_{t-1} + a_t$ is an outlier free series which follows the AR(1) model, $\omega$ is the outlier parameter, $\phi$ is the autoregressive parameter and $a_t$ is the error term with mean '0' and variance $\sigma_a^2$ and $P_t^{(T)} = 1$ if $t = T$ and $P_t^{(T)} = 0$ otherwise. The model can be written as

$$Y_t = \omega P_t^{(T)} + \phi Z_{t-1} + a_t \tag{3.2.2}$$

Hence, the AO model can be represented as

$$\begin{aligned} Y_t &= \omega + \phi Z_{t-1} + a_t && , t = T \\ &= \phi Z_{t-1} + a_t && , t \neq T \end{aligned} \tag{3.2.3}$$

Similarly, to study the effect of IO type outlier on the observed series, we consider the IO outlier model

$$Y_t = \omega \psi(B) P_t^{(T)} + Z_t \tag{3.2.4}$$

where $Z_t$ follows the AR(1) model in Equation (3.2.1), $\omega$ is the outlier parameter and $P_t^{(T)}$ is an indicator function as mentioned above. This model can be expressed as

$$Y_t = \omega \psi(B) P_t^{(T)} + \phi Z_{t-1} + a_t \tag{3.2.5}$$

which can alternatively be written as

$$\begin{aligned} Y_t &= \phi Z_{t-1} + a_t && , t < T \\ &= \omega \psi_{t-T} + \phi Z_{t-1} + a_t && , t \geq T \end{aligned} \tag{3.2.6}$$

where $\Psi_j = \phi^j$ for AR(1) model.

In order to study the effect of outliers in the AR(1) model, one particular generated outlier free series along with the corresponding outlier contaminated series with AO and IO outlier at $t = 50$ for $\omega = 10$ are plotted and it shows how the effect of

AO is 'local' at t = 50 whereas IO affects the observation at t = 50 and the succeeding observations as well.

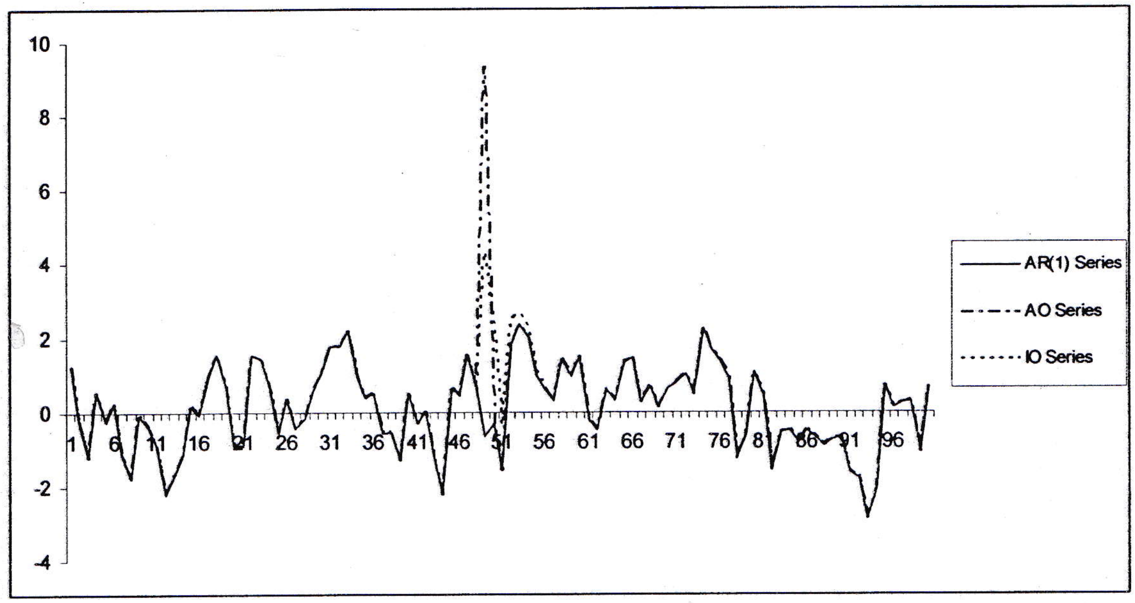


**Figure 3.1 Generated AR(1) Series with an Outlier at t = 50**

**(n = 100, $\phi$ = 0.5, $\mu$ = 0, $\sigma_a^2$=1, $\omega$ = 10)**

For a particular generated AR (1) model with n = 100, $\phi$ = 0.5, $\mu = 0$, $\sigma_a^2 = 1$ and $\omega$ = 0, 1, . . . ,10, based on 1000 replications, the average of mean and variance for outlier free series ($\omega$ = 0), AO and IO series at t = 50 are obtained as follow.

**Table (3.1)**

**Average of Mean and Variance for AR(1) Model with an AO and an IO at t=50**

**(n=100, $\phi$ = 0.5, $\mu$ = 0, $\sigma_a^2$=1; 1000 replications)**

| $\omega$ | AO | | IO | |
|---|---|---|---|---|
| | Mean | Variance | Mean | Variance |
| 0 | 0.0029 | 1.3094 | 0.0029 | 1.3094 |
| 1 | 0.0184 | 1.3109 | 0.0284 | 1.3148 |
| 2 | 0.0191 | 1.3579 | 0.0391 | 1.3691 |
| 3 | 0.0294 | 1.3941 | 0.0594 | 1.4215 |
| 4 | 0.0335 | 1.4520 | 0.0735 | 1.4984 |
| 5 | 0.0505 | 1.5571 | 0.1005 | 1.6355 |
| 6 | 0.0632 | 1.6754 | 0.1232 | 1.7874 |
| 7 | 0.0739 | 1.7935 | 0.1439 | 1.9433 |
| 8 | 0.0858 | 1.9337 | 0.1658 | 2.1319 |
| 9 | 0.0949 | 2.1079 | 0.1849 | 2.3532 |
| 10 | 0:1025 | 2.3129 | 0.2025 | 2.6056 |

In order to compare the changes in the estimates due to the presence of outlier of either type, the relative change (RC) is also computed for each estimate, where RC is defined as

$$\text{Relative Change (RC)} = \frac{A_\omega - A_0}{A_0} \times 100$$

where $A_\omega$ is the average estimated value at $\omega > 0$ (series with an outlier) and $A_0$ is the average estimated value at $\omega = 0$ (outlier free series).

The following table shows that the relative changes (RC) of each estimated value of mean and variance for the generated AR(1) series with an AO and an IO at $t = 50$ for $\omega = 0, 1, \ldots, 10$, $\mu = 0$, $\sigma_a^2 = 1$ and $\phi = 0.5$.

**Table (3.2)**

**Relative Changes (%) for Average of Mean and Variance for AR(1) Model with an AO and an IO at $t = 50$ ($n=100$, $\phi = 0.5$, $\mu = 0$, $\sigma_a^2 = 1$; 1000 replications)**

| ω | AO | | IO | |
|---|---|---|---|---|
| | RC for Mean | RC for Variance | RC for Mean | RC for Variance |
| 1 | 534.4828 | 0.1146 | 879.3103 | 0.4124 |
| 2 | 558.6207 | 3.7040 | 1248.2759 | 4.5593 |
| 3 | 913.7931 | 6.4686 | 1948.2759 | 8.5612 |
| 4 | 1055.1724 | 10.8905 | 2434.4828 | 14.4341 |
| 5 | 1641.3793 | 18.9171 | 3365.5172 | 24.9045 |
| 6 | 2079.3103 | 27.9517 | 4148.2759 | 36.5053 |
| 7 | 2448.2759 | 36.9711 | 4862.0690 | 48.4115 |
| 8 | 2858.6207 | 47.6783 | 5617.2414 | 62.8150 |
| 8 | 3172.4138 | 60.9821 | 6275.6821 | 79.7159 |
| 10 | 3434.4828 | 76.6382 | 6882.7586 | 98.9919 |

According to Table (3.1), it is observed that the average of the mean and variance for the outlier free series ($\omega = 0$) are 0.0029 and 1.3094 respectively. From Tables (3.1) and (3.2) it is clear that an outlier of either type in the series has effect on the mean and variance and the inflated values of the mean and variance are obtained with increasing magnitude of outlier parameter $\omega$, though the effect of IO on both the mean and variance of the series is much more prominent for AR(1) series.

Suppose that the observed series $Y_t$ follows MA(1) model with a single outlier AO type at T, which is modeled as

$$Y_t = \omega P_t^{(T)} + Z_t \qquad (3.2.7)$$

where $Z_t = a_t - \theta\, a_{t-1}$ is the MA(1) model for outlier free series, ω is the outlier parameter, θ is the moving average parameter and $a_t$ is the error term with mean '0' and variance $\sigma_a^2$ and $P_t^{(T)} = 1$ if t = T and $P_t^{(T)} = 0$ otherwise. The model (3.2.7) can be written as

$$Y_t = \omega P_t^{(T)} + a_t - \theta a_{t-1} \tag{3.2.8}$$

It can be simplified to

$$\begin{aligned} Y_t &= \omega + a_t - \theta\, a_{t-1} && , t = T \\ &= a_t - \theta\, a_{t-1} && , t \neq T \end{aligned} \tag{3.2.9}$$

Similarly, the MA(1) model with an IO type outlier is considered as

$$Y_t = \omega \psi(B) P_t^{(T)} + a_t - \theta a_{t-1} \tag{3.2.10}$$

giving,

$$\begin{aligned} Y_t &= a_t - \theta\, a_{t-1} && , t < T \\ &= \omega + a_t - \theta\, a_{t-1} && , t = T \\ &= \omega\, \Psi_1 + a_t - \theta\, a_{t-1} && , t = T + 1 \\ &= a_t - \theta\, a_{t-1} && , t > T + 1 \end{aligned} \tag{3.2.11}$$

where $\Psi_1 = -\theta$.

Analogous to AR(1), one particular generated outlier free series of MA(1) and the corresponding outlier contaminated series for AO and IO at t = 50 with ω =10 are plotted in Figure 3.2.

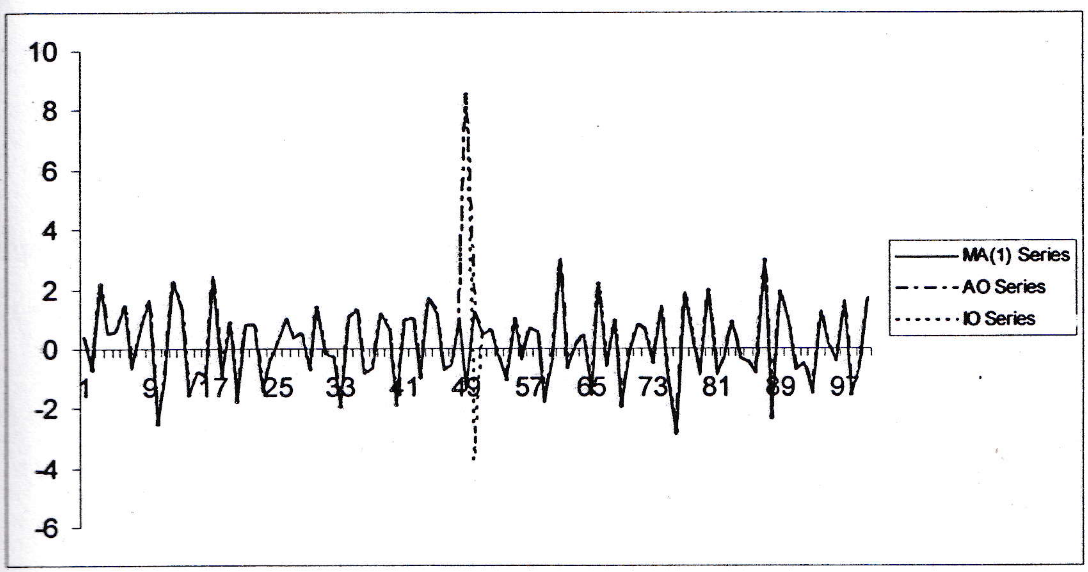


**Figure 3.2 Generated MA(1) Series with an Outlier at t = 50**

**(n = 100, θ = – 0.3, μ = 0, $\sigma_a^2$=1, ω = 10)**

Figure 3.2 shows that an AO outlier affects the series at time point t = 50 only. An IO affects the series at t = 50 and its decaying effect can be seen at t = 51 as well. Since the weights $\Psi_j$ of MA(1) are equal to zero when j > 1, there is no IO effect after t = 51.

As in AR(1) model, for a particular generated MA(1) model with n = 100, $\theta = -0.3$, $\mu = 0$, $\sigma_a^2 = 1$ and $\omega = 0, 1, \ldots, 10$, based on 1000 replications, the average of mean and variance for outlier free series ($\omega = 0$), AO and IO series at t = 50 are as below.

**Table (3.3)**

**Average of Mean and Variance for MA(1) Model with an AO and an IO at t=50**

**(n=100, $\theta = -0.3$, $\mu = 0$, $\sigma_a^2 = 1$; 1000 replications)**

| ω | AO | | IO | |
|---|---|---|---|---|
| | Mean | Variance | Mean | Variance |
| 0 | –0.0005 | 1.0826 | –0.0005 | 1.0826 |
| 1 | 0.0071 | 1.0946 | 0.0048 | 1.0959 |
| 2 | 0.0189 | 1.1207 | 0.0143 | 1.1251 |
| 3 | 0.0357 | 1.1717 | 0.0288 | 1.1807 |
| 4 | 0.0394 | 1.2484 | 0.0302 | 1.2656 |
| 5 | 0.0575 | 1.3323 | 0.0459 | 1.3592 |
| 6 | 0.0607 | 1.4560 | 0.0468 | 1.4921 |
| 7 | 0.0717 | 1.5661 | 0.0555 | 1.6169 |
| 8 | 0.0825 | 1.7117 | 0.0641 | 1.7779 |
| 9 | 0.0862 | 1.8969 | 0.0654 | 1.9818 |
| 10 | 0.0979 | 2.0779 | 0.0748 | 2.1824 |

The following table shows that the relative changes (RC) of each estimated value of mean and variance for the generated MA(1) series with an AO and an IO at t = 50 for $\omega = 0, 1, \ldots, 10$, $\mu = 0$, $\sigma_a^2 = 1$ and $\theta = -0.3$.

**Table (3.4)**

**Relative Changes (%) for Average of Mean and Variance for MA(1) Model with an AO and an IO at t = 50 (n=100, θ = –0.3, μ = 0, $\sigma_a^2$=1; 1000 replications)**

| ω | AO | | IO | |
|---|---|---|---|---|
| | RC for Mean | RC for Variance | RC for Mean | RC for Variance |
| 1 | -1520 | 1.1084 | -1060 | 1.2285 |
| 2 | -3880 | 3.5193 | -2960 | 3.9257 |
| 3 | -7240 | 8.2302 | -5860 | 9.0615 |
| 4 | -7980 | 15.3150 | -6140 | 16.9038 |
| 5 | -11600 | 23.0648 | -9280 | 25.5496 |
| 6 | -12240 | 34.4910 | -9460 | 37.8256 |
| 7 | -14440 | 44.6610 | -11200 | 49.3534 |
| 8 | -16600 | 58.1101 | -12920 | 64.2250 |
| 8 | -17340 | 75.2171 | -13180 | 83.0593 |
| 10 | -19680 | 91.9361 | -15060 | 101.5888 |

According to Table (3.3), it can be seen that the average of the mean and variance for the outlier free series (ω = 0) are -0.0005 and 1.0826 respectively. From Tables (3.3) and (3.4), it is observed that an outlier of either type in the series has effect on the mean and variance and the higher values of mean and variance are found with increasing magnitude of outlier parameter ω, though the effect of AO on the mean of the series is much more prominent but the effect of IO on variance of the series is much more prominent for MA(1) series.

Again, the observed series $Y_t$ with ARMA(1,1) model is now considered and a single AO type outlier at T is given by

$$Y_t = \omega P_t^{(T)} + Z_t \qquad (3.2.12)$$

where $Z_t = \phi\, Z_{t-1} + a_t - \theta\, a_{t-1}$ is the ARMA(1,1) model which is an outlier free series, ω is the outlier parameter, φ and θ are the autoregressive and moving average parameters respectively and $a_t$ is the error term with mean '0' and variance $\sigma_a^2$ and $P_t^{(T)} = 1$ if $t = T$ and $P_t^{(T)} = 0$ elsewhere. The model can be described as

$$Y_t = \omega P_t^{(T)} + \phi Z_{t-1} + a_t - \theta\, a_{t-1} \qquad (3.2.13)$$

Hence, the AO model can be represented as

$$\begin{aligned} Y_t &= \omega + \phi\, Z_{t-1} + a_t - \theta\, a_{t-1} &&, t = T \\ &= \phi\, Z_{t-1} + a_t - \theta\, a_{t-1} &&, t \neq T \end{aligned} \qquad (3.2.14)$$

Similarly, to study the effect of IO type outlier on the observed series, the IO outlier model

$$Y_t = \omega\psi(B)P_t^{(T)} + Z_t \qquad (3.2.15)$$

is considered. Here, $Z_t$ follows the ARMA(1,1) model in Equation (3.2.12), $\omega$ is the outlier parameter and $P_t^{(T)}$ is an indicator function as mentioned above. This model can be represented as

$$Y_t = \omega\psi(B)P_t^{(T)} + \phi Z_{t-1} + a_t - \theta\, a_{t-1} \qquad (3.2.16)$$

which can alternatively be written as

$$\begin{aligned} Y_t &= \phi\, Z_{t-1} + a_t - \theta\, a_{t-1} && , t < T \\ &= \omega\psi_{t-T} + \phi\, Z_{t-1} + a_t - \theta\, a_{t-1} && , t \geq T \end{aligned} \qquad (3.2.17)$$

where $\Psi_j = \phi^{j-1}(\phi - \theta)$ for ARMA(1,1) model.

In order to study the effect of outliers in the ARMA(1,1) model, one particular generated outlier free ARMA(1,1) series and the corresponding outlier contaminated series for AO and IO outliers at t = 50 for $\omega$ = 10 are plotted and it shows that the effect of AO is at t = 50 only whereas IO affects the observation at t = 50 and the succeeding observations as well.

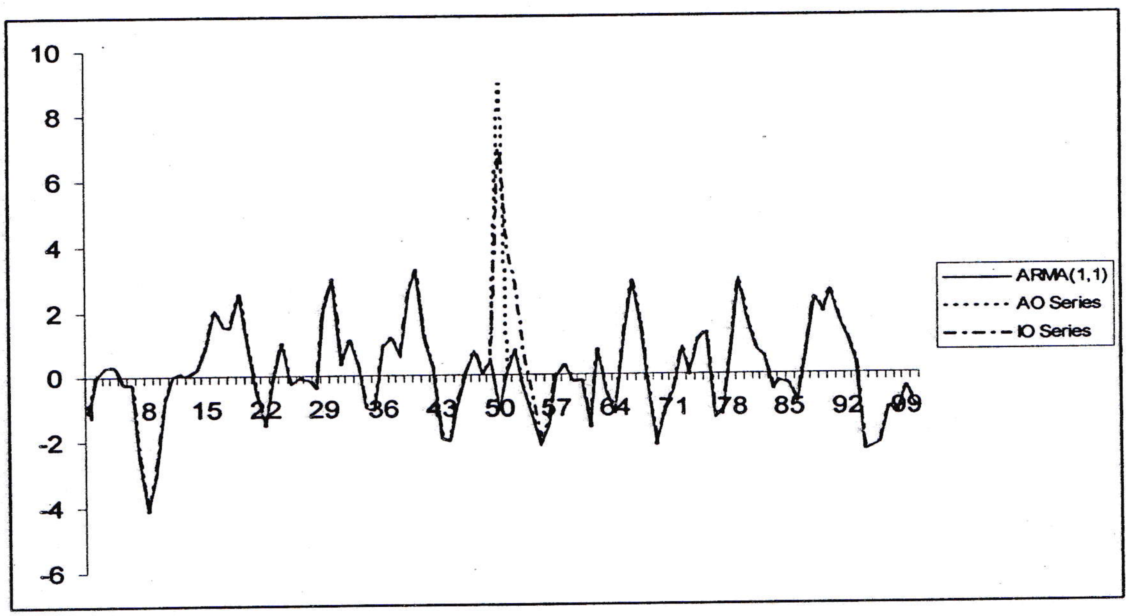


**Figure 3.3 Generated ARMA(1,1) Series with an Outlier at t = 50 ($n = 100, \phi = 0.5, \theta = -0.3, \mu = 0, \sigma_a^2 = 1, \omega = 10$)**

For a particular generated ARMA(1,1) model with n = 100, $\phi$ = 0.5, $\theta = -0.3$, $\mu = 0$, $\sigma_a^2 = 1$ and $\omega$ = 0, 1, . . . , 10, based on 1000 replications, the average values of mean and variance for outlier free series ($\omega = 0$), AO and IO series at $t = 50$ are obtained as follow.

**Table (3.5)**

**Average of Mean and Variance for ARMA(1,1) Model with an AO and an IO at t=50**

**(n=100, $\phi = 0.5$, $\theta = -0.3$, $\mu = 0$, $\sigma_a^2 = 1$; 1000 replications)**

| $\omega$ | AO | | IO | |
|---|---|---|---|---|
| | Mean | Variance | Mean | Variance |
| 0 | 0.0064 | 1.8019 | 0.0064 | 1.8019 |
| 1 | 0.0072 | 1.8081 | 0.0292 | 1.8340 |
| 2 | 0.0194 | 1.8443 | 0.0634 | 1.9378 |
| 3 | 0.0230 | 1.8851 | 0.0890 | 2.1006 |
| 4 | 0.0487 | 1.9525 | 0.1367 | 2.2363 |
| 5 | 0.0490 | 2.0423 | 0.1590 | 2.6253 |
| 6 | 0.0678 | 2.1630 | 0.1998 | 3.0124 |
| 7 | 0.0696 | 2.3068 | 0.2209 | 3.4517 |
| 8 | 0.0759 | 2.4458 | 0.2519 | 3.9238 |
| 9 | 0.0864 | 2.6290 | 0.2844 | 4.5388 |
| 10 | 0.0962 | 2.7934 | 0.3162 | 5.1192 |

The following table shows that the relative changes (RC) of each estimated value of mean and variance for the generated ARMA(1, 1) series with an AO and an IO at $t = 50$ for $\omega = 0, 1, \ldots, 10$, $\mu = 0$, $\sigma_a^2 = 1$, $\phi = 0.5$ and $\theta = -0.3$.

**Table (3.6)**

**Relative Changes (%) for Average of Mean and Variance for ARMA(1, 1) Model with an AO and an IO at t = 50**

**(n=100, $\phi = 0.5$, $\theta = -0.3$, $\mu = 0$, $\sigma_a^2 = 1$; 1000 replications)**

| $\omega$ | AO | | IO | |
|---|---|---|---|---|
| | RC for Mean | RC for Variance | RC for Mean | RC for Variance |
| 1 | 12.5000 | 0.3441 | 356.2500 | 1.7815 |
| 2 | 203.1250 | 2.3531 | 890.6250 | 7.5420 |
| 3 | 259.3750 | 4.6173 | 1290.6250 | 16.5769 |
| 4 | 660.9375 | 8.3578 | 2035.9375 | 24.1079 |
| 5 | 665.6250 | 13.3415 | 2384.3750 | 45.6962 |
| 6 | 959.3750 | 20.0400 | 3021.8750 | 67.1791 |
| 7 | 987.5000 | 28.0204 | 3351.5625 | 91.5589 |
| 8 | 1085.9375 | 35.7345 | 3835.9375 | 117.7590 |
| 8 | 1250.0000 | 49.9015 | 4343.7500 | 151.8897 |
| 10 | 1403.1250 | 55.0253 | 4840.6250 | 184.1001 |

According to Table (3.5), it is observed that the average of the mean and variance for the outlier free series ($\omega = 0$) are 0.0064 and 1.8019 respectively. From Tables (3.5) and (3.6) it is noticed that an outlier of both types in the series have effect on the mean and variance and the inflated values of mean and variance are resulted with increasing magnitude of outlier parameter $\omega$, though the effect of IO on both the mean and variance of the series is much more prominent for ARMA(1,1) series as in the case of AR(1).

Based on the simulated results, the effect of IO on the mean is much more significant for both AR(1) and ARMA(1,1) series but the effect of AO on the mean is much more prominent for MA(1) series. In addition, it is also clear that the effect of IO on the variance is much more obvious than the effect of AO for both AR(1) and MA(1) as well as ARMA(1,1) cases. Moreover, it can be concluded that the effect of outlier on mean and variance depends on both magnitude and type of outlier.

## 3.3 Effects on Autocorrelation and Partial Autocorrelation Functions

For each of the generated AR(1) and MA(1) outlier free series, together with their respective AO and IO series, the lag-1 autocorrelation coefficient $\rho_1$ and the lag-1 partial autocorrelation coefficient $\phi_{11}$ were estimated. Since the values of $\rho_1$ and $\phi_{11}$ are identical in both AR(1) and MA(1) cases, it only needs to study the effect of outliers on the autocorrelation function.

The following table represents the average of estimated values of $\rho_1$ for the generated AR(1) outlier free series ($\omega = 0$) and the corresponding outlier contaminated series with an AO and an IO outliers at $t = 50$ for $\omega = 0, 1, \ldots, 10$, $\mu = 0$, $\sigma_a^2 = 1$ and $\phi = 0.5$.

**Table (3.7)**

**Average of Estimated Values of $\rho_1$ : AR(1) with an AO and an IO at t=50**

**(n=100, $\phi = 0.5$, $\mu = 0$, $\sigma_a^2$=1; 1000 replications)**

| ω | Estimated Values of $\rho_1$ | |
|---|---|---|
| | AO | IO |
| 0 | 0.4702 | 0.4702 |
| 1 | 0.4664 | 0.4703 |
| 2 | 0.4539 | 0.4686 |
| 3 | 0.4361 | 0.4673 |
| 4 | 0.4178 | 0.4711 |
| 5 | 0.3964 | 0.4761 |
| 6 | 0.3635 | 0.4706 |
| 7 | 0.3373 | 0.4720 |
| 8 | 0.3104 | 0.4720 |
| 9 | 0.2866 | 0.4747 |
| 10 | 0.2632 | 0.4765 |

The following table shows that the relative changes (RC) of each estimated value of lag-1 autocorrelation $\rho_1$ for the generated AR (1) series with an AO and an IO at $t = 50$ for $\omega = 0, 1, \ldots, 10$, $\mu = 0$, $\sigma_a^2 = 1$ and $\phi = 0.5$.

**Table (3.8)**

**Relative Changes (%) for Average of Estimated Values of $\rho_1$ : AR(1) with an AO and an IO at t = 50 (n=100, $\phi = 0.5$, $\mu = 0$, $\sigma_a^2$=1; 1000 replications)**

| ω | RC for Estimated Values of $\rho_1$ | |
|---|---|---|
| | AO | IO |
| 1 | -0.8082 | 0.0213 |
| 2 | -3.4666 | -0.3403 |
| 3 | -7.2522 | -0.6168 |
| 4 | -11.1442 | 0.1914 |
| 5 | -15.6954 | 1.2548 |
| 6 | -22.6925 | 0.0851 |
| 7 | -28.2646 | 0.3828 |
| 8 | -33.9855 | 0.3828 |
| 9 | -39.0472 | 0.9570 |
| 10 | -44.0238 | 1.3399 |

According to Table (3.7), it is observed that the average of estimated value of lag-1 autocorrelation coefficient $\rho_1$ for the outlier free series ($\omega = 0$) is 0.4702. From

Tables (3.7) and (3.8), it can be seen that the estimated values of $\rho_1$ do not change significantly with the increasing values outlier $\omega$ for IO case of AR(1) series. However, the estimated value of lag-1 autocorrelation $\rho_1$ decreases as the value of outlier parameter $\omega$ increases for AO case of AR(1) series. It is clear that the effect of outlier on the autocorrelation function is much more significant in the case of AO than that of IO for AR(1) series. The same conclusion can be made for the effect of outlier on the partial autocorrelation function $\phi_{11}$ since the values of both $\rho_1$ and $\phi_{11}$ are equal for AR(1) series.

The following table shows the average of estimated values of $\rho_1$ for the generated MA(1) outlier free series ($\omega = 0$) and the corresponding outlier contaminated series with an AO and an IO outlier at $t = 50$ for $\omega = 0, 1, \ldots, 10$ and $\theta = -0.3$, based on 1000 replications.

**Table (3.9)**

**Average of Estimated Values of $\rho_1$ : MA(1) with an AO and an IO at t=50**

**(n=100, $\theta$ = -0.3, $\mu = 0$, $\sigma_a^2$=1; 1000 replications)**

| $\omega$ | Estimated Values of $\rho_1$ | |
|---|---|---|
| | AO | IO |
| 0 | 0.2462 | 0.2462 |
| 1 | 0.2605 | 0.2628 |
| 2 | 0.2502 | 0.2594 |
| 3 | 0.2395 | 0.2595 |
| 4 | 0.2321 | 0.2662 |
| 5 | 0.2294 | 0.2828 |
| 6 | 0.1994 | 0.2658 |
| 7 | 0.1577 | 0.2438 |
| 8 | 0.1563 | 0.2581 |
| 9 | 0.1550 | 0.2707 |
| 10 | 0.1215 | 0.2555 |

The following table shows that the relative changes (RC) of each estimated value of lag-1 autocorrelation $\rho_1$ for the generated MA (1) series with an AO and an IO at $t = 50$ for $\omega = 0, 1, \ldots, 10$, $\mu = 0$, $\sigma_a^2 = 1$ and $\theta = -0.3$.

**Table (3.10)**

**Relative Changes (%) for Average of Estimated Values of $\rho_1$ : MA(1) with an AO and an IO at t = 50 (n=100, $\theta$ = -0.3, $\mu$ = 0, $\sigma_a^2$=1; 1000 replications)**

| ω | RC for Estimated Values of $\rho_1$ | |
|---|---|---|
| | AO | IO |
| 1 | 5.8083 | 6.7425 |
| 2 | 1.6247 | 5.3615 |
| 3 | -2.7214 | 5.4021 |
| 4 | -5.7271 | 8.1235 |
| 5 | -6.8237 | 14.8660 |
| 6 | -19.0089 | 7.9610 |
| 7 | -35.9464 | -0.9748 |
| 8 | -36.5150 | 4.8335 |
| 9 | -37.0431 | 9.9513 |
| 10 | -50.6499 | 3.7774 |

According to Table (3.9), it can be seen that the average of estimated value of lag-1 autocorrelation $\rho_1$ for the outlier free series ($\omega$ = 0) is 0.2462. According to Tables (3.9) and (3.10), it is evident that the effect of outlier on the autocorrelation function is much more obvious in the case of AO than that of IO for MA(1) series as in the case of AR(1). For MA(1) series, it can also be seen that the estimated values of $\rho_1$ do not change significantly with the increasing values of outlier parameter $\omega$ for IO case. But, the higher the values of the outlier parameter $\omega$, the larger the effect of AO outlier on MA(1) series. The same conclusion can be made for the effect of outlier on the partial autocorrelation function since the values of both $\rho_1$ and $\phi_{11}$ are also identical for MA(1) series.

For each of the generated ARMA(1, 1) outlier free series ($\omega$ = 0), together with their respective AO and IO series, the lag-1 autocorrelation coefficient $\rho_1$ and the lag-1 partial autocorrelation coefficient $\phi_{11}$ were estimated. Since the values of $\rho_1$ and $\phi_{11}$ are identical in ARMA(1, 1) case as in the both AR(1) and MA(1) cases, it is enough to study the effect of outliers on the autocorrelation function only.

The following table represents the average of estimated values of $\rho_1$ for the generated ARMA(1,1) outlier free series ($\omega$ = 0) and the corresponding outlier contaminated series with an AO and an IO outliers at t = 50 for $\omega$ = 0, 1, . . . , 10, $\mu$ = 0, $\sigma_a^2$=1, $\phi$ = 0.5 and $\theta$ = –0.3, based on 1000 replications.

**Table (3.11)**

**Average of Estimated Values of $\rho_1$ : ARMA(1,1) with an AO and an IO at t = 50**

**(n=100, $\phi$ = 0.5, $\theta$ = -0.3, $\mu$ = 0, $\sigma_a^2$=1; 1000 replications)**

| ω | Estimated Values of $\rho_1$ | |
|---|---|---|
| | AO | IO |
| 0 | 0.6329 | 0.6329 |
| 1 | 0.6365 | 0.6385 |
| 2 | 0.6118 | 0.6141 |
| 3 | 0.6056 | 0.6121 |
| 4 | 0.5737 | 0.5969 |
| 5 | 0.5527 | 0.5826 |
| 6 | 0.5166 | 0.5697 |
| 7 | 0.5040 | 0.5621 |
| 8 | 0.4456 | 0.5377 |
| 9 | 0.4121 | 0.5333 |
| 10 | 0.3977 | 0.5303 |

The following table shows that the relative changes (RC) of each estimated value of lag-1 autocorrelation $\rho_1$ for the generated ARMA(1, 1) series with an AO and an IO at t = 50 for $\omega = 0, 1, \ldots, 10$, $\mu = 0$, $\sigma_a^2 = 1$, $\phi = 0.5$ and $\theta = -0.3$.

**Table (3.12)**

**Relative Changes (%) for Average of Estimated Values of $\rho_1$ :**

**ARMA(1,1) with an AO and an IO at t = 50**

**(n=100, $\phi$ = 0.5, $\theta$ = -0.3, $\mu$ = 0, $\sigma_a^2$=1; 1000 replications)**

| ω | RC for Estimated Values of $\rho_1$ | |
|---|---|---|
| | AO | IO |
| 1 | 0.5688 | 0.8848 |
| 2 | -3.3339 | -2.9705 |
| 3 | -4.3135 | -3.2865 |
| 4 | -9.3538 | -5.6881 |
| 5 | -12.6718 | -7.9475 |
| 6 | -18.3757 | -9.9858 |
| 7 | -20.3666 | -11.1866 |
| 8 | -29.5939 | -15.0419 |
| 9 | -34.8870 | -15.7371 |
| 10 | -37.1623 | -16.2111 |

According to Table (3.11), it is obtained that the average of estimated value of lag-1 autocorrelation $\rho_1$ for the outlier free series ($\omega$ = 0) is 0.6329. From Tables (3.11) and (3.12), it can also be seen that the estimated value of lag-1 autocorrelation $\rho_1$ decreases as the value of outlier parameter $\omega$ increases for both AO and IO cases of ARMA(1,1) series. However, it is clear that the effect of outliers on the autocorrelation function is much more significant in the case of AO than that of IO for ARMA(1,1) series as in both AR(1) and MA(1) series. The same conclusion can be made for the effect of outlier on the partial autocorrelation function since the values of both $\rho_1$ and $\phi_{11}$ are equal for ARMA(1,1) case.

From the simulated results, it is clear that the effect of presence of outlier on the estimates of autocorrelation and partial autocorrelation functions depend on both the magnitude and type of outlier for the underlying models of AR(1), MA(1) and ARMA(1,1).

## 3.4 Effects on Parameter Estimates and Error Variances

For each of the generated outlier free AR(1), MA(1) and ARMA(1,1) series along with their corresponding outlier contaminated series with AO and IO outliers, the parameters of interest, namely the time series parameters $\phi$ for AR(1), $\theta$ for MA(1), both $\phi$ and $\theta$ for ARMA(1,1), as well as the error variance $\sigma_a^2$ for each series were estimated.

The table below represents the estimated values of $\phi$ and $\sigma_a^2$ for the generated AR(1) outlier free ($\omega$ = 0) series and the corresponding outlier contaminated series with an AO and an IO at t = 50 for $\omega$ = 0, 1, . . . , 10, $\mu$ = 0, $\sigma_a^2$=1 and $\phi$ = 0.5, based on 1000 replications.

**Table (3.13)**

**Average of Estimated Values of $\phi$ and $\sigma_a^2$: AR(1) with an AO and an IO at t=50 (n=100, $\phi = 0.5$, $\mu = 0$, $\sigma_a^2$=1; 1000 replications)**

| ω | Estimated Value of $\phi$ | | Estimated Value of $\sigma_a^2$ | |
|---|---|---|---|---|
| | AO | IO | AO | IO |
| 0 | 0.4899 | 0.4899 | 0.9909 | 0.9909 |
| 1 | 0.4874 | 0.4913 | 0.9991 | 0.9968 |
| 2 | 0.4747 | 0.4895 | 1.0390 | 1.0290 |
| 3 | 0.4561 | 0.4876 | 1.0927 | 1.0727 |
| 4 | 0.4387 | 0.4918 | 1.1893 | 1.1543 |
| 5 | 0.4155 | 0.4953 | 1.2931 | 1.2245 |
| 6 | 0.3820 | 0.4892 | 1.4302 | 1.3606 |
| 7 | 0.3548 | 0.4904 | 1.5594 | 1.4716 |
| 8 | 0.3285 | 0.4910 | 1.7477 | 1.6459 |
| 9 | 0.3044 | 0.4933 | 1.9324 | 1.8091 |
| 10 | 0.2807 | 0.4947 | 2.1386 | 2.0004 |

The following table shows that the relative changes (RC) of each estimated parameter $\phi$ and $\sigma_a^2$ for the generated AR(1) series with an AO and an IO at $t = 50$ for $\omega = 0, 1, \ldots, 10$, $\mu = 0$, $\sigma_a^2$=1 and $\phi = 0.5$.

**Table (3.14)**

**Relative Changes (%) for Average of Estimated Values of $\phi$ and $\sigma_a^2$: AR(1) with an AO and an IO at t = 50 (n=100, $\phi = 0.5$, $\mu = 0$, $\sigma_a^2$=1; 1000 replications)**

| ω | RC for Estimated Value of $\phi$ | | RC for Estimated Value $\sigma_a^2$ | |
|---|---|---|---|---|
| | AO | IO | AO | IO |
| 1 | -0.51 | 0.29 | 0.83 | 0.60 |
| 2 | -3.10 | -0.08 | 4.85 | 3.84 |
| 3 | -6.90 | -0.47 | 10.27 | 8.26 |
| 4 | -10.45 | 0.39 | 20.02 | 16.49 |
| 5 | -15.19 | 1.10 | 30.50 | 23.57 |
| 6 | -22.02 | -0.14 | 44.33 | 37.31 |
| 7 | -27.58 | 0.10 | 57.37 | 48.51 |
| 8 | -32.95 | 0.22 | 76.38 | 66.10 |
| 9 | -37.86 | 0.69 | 95.01 | 82.57 |
| 10 | -42.70 | 0.98 | 115.82 | 101.88 |

From Table (3.13), it is observed that the average of estimated values of $\phi$ and $\sigma_a^2$ for the generated AR(1) outlier free ($\omega = 0$) series are 0.4899 and 0.9909 respectively. According to Tables (3.13) and (3.14), the estimated values of $\phi$ do not change significantly with varying outlier parameter ω in the case of IO as compared to

AO for AR(1) series. In the case of AO, estimated values of parameter $\phi$ decrease with the increasing magnitude of outlier parameter $\omega$ that has been found as in estimated autocorrelation values for AR(1) series. The effect of outlier on the estimated values of $\phi$ is less for IO. It can also be seen from Tables (3.13) and (3.14) that with increasing magnitude of outlier parameter $\omega$, the estimate of all parameters change, though the effect on the estimate of the error variance $\sigma_a^2$ is much more prominent and the error variance tends to be overestimated more and more with higher values of outlier parameter $\omega$ in both AO and IO cases. This overestimation happens irrespective of the type of outlier, though the overestimation is more prominent in the case of AO for AR(1) series. It is clear from the simulated results that the effect of the presence of outlier of either type on the estimate of error variance $\sigma_a^2$ is much more than that on the estimate of $\phi$.

The table below shows the average of estimated values of $\theta$ and $\sigma_a^2$ for the generated MA(1) outlier free series ($\omega = 0$) and corresponding outlier contaminated series with an AO and an IO at $t = 50$ for $\omega = 0, 1, \ldots, 10$, $\mu = 0$, $\sigma_a^2 = 1$ and $\theta = -0.3$, based on 1000 replications.

**Table (3.15)**

**Average of Estimated Values of $\theta$ and $\sigma_a^2$: MA(1) with an AO and an IO at t=50**

**(n=100, $\theta$ = - 0.3, $\mu = 0$, $\sigma_a^2$=1; 1000 replications)**

| $\omega$ | Estimated Value of $\theta$ | | Estimated Value of $\sigma_a^2$ | |
|---|---|---|---|---|
| | AO | IO | AO | IO |
| 0 | -0.2942 | -0.2942 | 1.0135 | 1.0135 |
| 1 | -0.2934 | -0.2948 | 1.0216 | 1.0208 |
| 2 | -0.2927 | -0.2930 | 1.0434 | 1.0412 |
| 3 | -0.2888 | -0.2992 | 1.0904 | 1.0810 |
| 4 | -0.2626 | -0.2961 | 1.2046 | 1.1910 |
| 5 | -0.2600 | -0.2902 | 1.2838 | 1.2612 |
| 6 | -0.2296 | -0.2961 | 1.3840 | 1.3556 |
| 7 | -0.1869 | -0.2925 | 1.4989 | 1.4640 |
| 8 | -0.1759 | -0.2992 | 1.6445 | 1.6054 |
| 9 | -0.1671 | -0.2977 | 1.8718 | 1.8167 |
| 10 | -0.1359 | -0.2935 | 2.0381 | 1.9863 |

The table below indicates the relative changes (RC) for estimated values of θ and $\sigma_a^2$ for the generated MA(1) outlier contaminated series with an AO and an IO at t = 50 for ω = 0, 1, . . . , 10, μ = 0, $\sigma_a^2$=1 and θ = –0.3, based on 1000 replications.

**Table (3.16)**
**Relative Changes (%) for Average of Estimated Values of θ and $\sigma_a^2$: MA(1) with an AO and an IO at t = 50 (n=100, θ = - 0.3, μ = 0, $\sigma_a^2$=1; 1000 replications)**

| ω | RC for Estimated Value of θ | | RC for Estimated Value $\sigma_a^2$ | |
|---|---|---|---|---|
| | AO | IO | AO | IO |
| 1 | -0.27 | 0.20 | 0.80 | 0.72 |
| 2 | -0.51 | -0.41 | 2.95 | 2.73 |
| 3 | -1.84 | 1.70 | 7.59 | 6.66 |
| 4 | -10.74 | 0.65 | 18.86 | 17.51 |
| 5 | -11.62 | -1.36 | 26.67 | 24.44 |
| 6 | -21.96 | 0.65 | 36.56 | 33.75 |
| 7 | -36.47 | -0.58 | 47.89 | 44.45 |
| 8 | -40.21 | 1.70 | 62.26 | 58.40 |
| 9 | -43.20 | 1.19 | 84.69 | 79.25 |
| 10 | -53.81 | -0.24 | 101.10 | 95.98 |

From Table (3.15), it can be seen that the average of estimated values of θ and $\sigma_a^2$ for the generated MA(1) outlier free (ω = 0) series are -0.2942 and 1.0135 respectively. According to Tables (3.15) and (3.16), the estimated values of θ do not differ much from the estimated value of θ without an outlier in the case of IO but show significant change in the case of AO for MA(1) series. The higher the value of ω, the lower the value of estimate of θ for AO case. It can also be seen that the estimate of error variance $\sigma_a^2$ increases with the value of ω, irrespective of the type of outlier and differ much more in AO than IO. Also, analogous to AR(1), AO shows higher impact on the estimate of error variance $\sigma_a^2$ than IO for MA(1). It is clear from the simulated results that the effect of the presence of outlier of either type on the estimate of error variance $\sigma_a^2$ is much more than that on the estimate of θ.

The table below represents the average of estimated values of ϕ, θ and $\sigma_a^2$ for the generated ARMA(1,1) outlier free series (ω = 0) and the corresponding outlier contaminated series with an AO and an IO at t = 50 for ω = 0, 1, . . . , 10, μ = 0, $\sigma_a^2$ = 1, ϕ = 0.5 and θ = –0.3, based on 1000 replications.

**Table (3.17)**

**Average of Estimated Values of $\phi$ and $\sigma_a^2$: ARMA(1,1) with an AO and an IO at t = 50 (n = 100, $\phi$ = 0.5, $\theta$ = –0.3, $\mu$ = 0, $\sigma_a^2$ = 1; 1000 replications)**

| $\omega$ | Estimated Value of $\phi$ | | Estimated Value of $\theta$ | | Estimated Value of $\sigma_a^2$ | |
|---|---|---|---|---|---|---|
| | AO | IO | AO | IO | AO | IO |
| 0 | 0.4912 | 0.4912 | -0.3161 | -0.3161 | 0.9788 | 0.9788 |
| 1 | 0.4954 | 0.4948 | -0.2973 | -0.3075 | 0.9910 | 0.9850 |
| 2 | 0.4872 | 0.4879 | -0.2742 | -0.3115 | 1.0374 | 1.0110 |
| 3 | 0.4897 | 0.4900 | -0.2335 | -0.3115 | 1.1270 | 1.0716 |
| 4 | 0.4890 | 0.4885 | -0.1701 | -0.3023 | 1.2260 | 1.1299 |
| 5 | 0.4950 | 0.4937 | -0.1243 | -0.3074 | 1.3584 | 1.2254 |
| 6 | 0.4879 | 0.4892 | -0.0830 | -0.3089 | 1.5089 | 1.3301 |
| 7 | 0.4951 | 0.4935 | -0.0200 | -0.3014 | 1.6732 | 1.4617 |
| 8 | 0.4919 | 0.4953 | 0.0138 | -0.3025 | 1.8600 | 1.6099 |
| 9 | 0.4943 | 0.4943 | 0.0581 | -0.3085 | 2.0598 | 1.7779 |
| 10 | 0.4969 | 0.4993 | 0.0993 | -0.3011 | 2.2864 | 1.9501 |

The table below represents the relative changes (RC) for estimated values of $\phi$, $\theta$ and $\sigma_a^2$ for the generated ARMA(1,1) outlier contaminated series with an AO and an IO at t = 50 for $\omega = 0, 1, \ldots, 10$, $\mu = 0$, $\sigma_a^2 = 1$, $\phi = 0.5$ and $\theta = -0.3$, based on 1000 replications.

**Table (3.18)**

**Relative Changes (%) for Average of Estimated Values of $\phi$ and $\sigma_a^2$: ARMA(1, 1) with an AO and an IO at t = 50**

**(n = 100, $\phi$ = 0.5, $\theta$ = –0.3, $\mu$ = 0, $\sigma_a^2$ = 1; 1000 replications)**

| $\omega$ | RC for Estimated Value of $\phi$ | | RC for Estimated Value of $\theta$ | | RC for Estimated Value of $\sigma_a^2$ | |
|---|---|---|---|---|---|---|
| | AO | IO | AO | IO | AO | IO |
| 1 | 0.86 | 0.73 | -5.95 | -2.72 | 1.25 | 0.63 |
| 2 | -0.81 | -0.67 | -13.26 | -1.46 | 5.99 | 3.39 |
| 3 | -0.31 | -0.24 | -26.13 | -1.46 | 15.14 | 9.48 |
| 4 | -0.45 | -0.55 | -46.19 | -4.37 | 25.26 | 15.44 |
| 5 | -0.77 | 0.51 | -60.68 | -2.75 | 38.78 | 25.19 |
| 6 | -0.67 | -0.41 | -73.74 | -2.28 | 54.16 | 35.89 |
| 7 | -0.79 | 0.47 | -93.67 | -4.65 | 71.97 | 49.34 |
| 8 | -0.14 | 0.83 | -104.37 | -4.30 | 90.03 | 64.48 |
| 9 | -0.63 | 0.63 | -118.38 | -2.40 | 110.44 | 81.64 |
| 10 | 1.16 | 1.65 | -131.41 | -4.75 | 133.69 | 99.23 |

From Table (3.17), it can be seen that the average of estimated values of $\phi$, $\theta$ and $\sigma_a^2$ for the generated ARMA(1,1) outlier free series ($\omega = 0$) are 0.4912, -0.3161 and 0.9788 respectively. According to Tables (3.17) and (3.18), the estimated values of $\phi$ do not differ significantly with varying values of $\omega$ for both AO and IO cases of ARMA(1,1) series. Moreover, the estimated values of $\theta$ also do not vary significantly with the increasing magnitude of outlier parameter $\omega$ in the IO case of ARMA(1,1) series. However, the estimated values of parameter $\theta$ decrease significantly with the increasing magnitude of outlier parameter $\omega$ in AO case for ARMA(1,1) series. It can also be seen from Table (3.11) and (3.12) that with increasing magnitude of outlier parameter $\omega$, the estimates of parameters ($\phi$ and $\theta$) change and the effect is larger for the estimated values of $\theta$ than that of $\phi$ in both AO and IO cases of ARMA(1,1) series. It is also found that the effect of outlier on the estimated values of $\theta$ is larger for AO than that of IO case. It can also be seen that the error variance tends to get overestimated significantly with higher values of outlier parameter $\omega$ in both cases of AO and IO for ARMA(1,1) series. This overestimation is irrespective of the type of outlier, though the overestimation is more prominent in case of AO for ARMA(1,1) series.

Based on the results presented, it is clear that for both AR(1) and MA(1) as well as ARMA(1,1), the estimate of error variance $\sigma_a^2$ is affected significantly by the presence of outlier and the effect increases with increasing values of outlier parameter $\omega$, irrespective of the type of outlier. In addition, the effect on the estimates of time series parameters as well as the error variance depends on both the magnitude and type of outlier.

It is clear that the presence of a single outlier of either type significantly overestimates the error variance. The finding provides empirical support to the claims in the literature that "the presence of outlier affects the estimate of error variance more than that of time series parameters" (Bruce and Martin, 1989; Ledolter, 1990).

This also supports the justification for using estimates of error variance for outlier detection procedure in addition to its usefulness in verifying model adequacy. Also the empirical study shows that though the presence of AO affects the estimate of error variance more than that of IO, the effect of IO on the estimate of error variance is also high and needs to be taken into account, contrary to some of the claims in the literature (Chang and Tiao, 1983; Bruce and Martin, 1989; Ljung, 1993).

### 3.5 Effects on AIC and BIC

In this section, the effect of an outlier on some model selection criteria based on residuals such as Akaike's Information Criterion (AIC) and Bayesian Information Criterion (BIC) are considered. Both AIC and BIC are monotone functions of the prediction error variance $\sigma_a^2$, usually estimated by the maximum likelihood (ML) estimate. The definitions of AIC and BIC are also presented in Appendix A. The commonly used ML-based AIC and BIC are affected by outliers in two ways: (a) they are distorted by grossly unreliable parameter estimates, and (b) they are greatly inflated by outliers (Martin, 1980).

For each of the generated outlier free ($\omega$ = 0) AR(1) series together their corresponding outlier contaminated series with AO and IO outliers, the values of AIC and BIC were estimated.

The following table shows the average of estimated values of AIC and BIC for the generated AR(1) outlier free series ($\omega$ = 0) and the corresponding outlier contaminated series with an AO and an IO at t = 50 for $\omega$ = 0, 1, . . . , 10, $\mu$ = 0, $\sigma_a^2$ = 1 and $\phi$ = 0.5, based on 1000 replications.

**Table (3.19)**

**Average of Estimated Values of AIC and BIC: AR(1) with an AO and an IO at t =50 (n=100, $\phi$ = 0.5, $\mu$ = 0, $\sigma_a^2$=1; 1000 replications)**

| ω | Estimated Value of AIC | | Estimated Value of BIC | |
|---|---|---|---|---|
| | AO | IO | AO | IO |
| 0 | 280.9652 | 280.9652 | 0.1290 | 0.1290 |
| 1 | 281.7932 | 281.5733 | 0.1373 | 0.1350 |
| 2 | 285.7621 | 284.8143 | 0.1764 | 0.1667 |
| 3 | 290.0750 | 288.9228 | 0.2268 | 0.2083 |
| 4 | 299.0750 | 296.1394 | 0.3115 | 0.2816 |
| 5 | 307.4578 | 303.6577 | 0.3952 | 0.3407 |
| 6 | 317.5019 | 312.4989 | 0.4960 | 0.4461 |
| 7 | 326.1275 | 320.3434 | 0.5825 | 0.5245 |
| 8 | 337.3607 | 331.4194 | 0.6965 | 0.6364 |
| 9 | 347.4249 | 340.6397 | 0.7969 | 0.7310 |
| 10 | 357.5022 | 350.8624 | 0.8983 | 0.8315 |

The table below represents the relative changes (RC) for estimated values of AIC and BIC for the generated AR(1) outlier contaminated series with an AO and an IO at t = 50 for ω = 0, 1, . . . , 10, μ = 0, $\sigma_a^2 = 1$ and ϕ = 0.5.

**Table (3.20)**

**Relative Changes (%) for Average of Estimated Values of AIC and BIC: AR(1) with an AO and an IO at t = 50 (n=100, ϕ = 0.5, μ = 0, $\sigma_a^2$=1; 1000 replications)**

| ω | RC for Estimated Value of AIC | | RC for Estimated Value of BIC | |
|---|---|---|---|---|
| | AO | IO | AO | IO |
| 1 | 0.2947 | 0.2164 | 6.4341 | 4.6512 |
| 2 | 1.7073 | 1.3700 | 36.7442 | 29.2248 |
| 3 | 3.2423 | 2.8322 | 75.8140 | 61.4729 |
| 4 | 6.4456 | 5.4007 | 141.4729 | 118.2946 |
| 5 | 9.4291 | 8.0766 | 206.3566 | 164.1085 |
| 6 | 13.0040 | 11.2233 | 284.4961 | 245.8140 |
| 7 | 16.0740 | 14.0153 | 351.5504 | 306.5891 |
| 8 | 20.0721 | 17.9575 | 439.9225 | 393.3333 |
| 9 | 23.6541 | 21.2391 | 517.7519 | 466.6667 |
| 10 | 27.2407 | 24.8775 | 596.3566 | 544.5736 |

According to Table (3.19), it can be seen that the average of estimated values of AIC and BIC for the generated AR(1) outlier free series (ω = 0) are 280.9652 and 0.1290 respectively. From Tables (3.19) and (3.20), it is clear that the effect of an outlier on the AIC and BIC is much more significant in the case of AO than that of IO for AR(1) series. It is also noticed that the inflated values of AIC and BIC are obtained when the values of outlier parameter ω increase.

Again, the following table represents the average of estimated values of AIC and BIC for the MA(1) outlier free series (ω = 0) and the corresponding outlier contaminated series with an AO and an IO at t = 50 for ω = 0, 1, 2, . . . , 10, μ = 0, $\sigma_a^2 = 1$ and θ = −0.3, based on 1000 replications.

**Table (3.21)**

**Average of Estimated Values of AIC and BIC: MA(1) with an AO and an IO at t = 50 (n=100, θ = –0.3, μ = 0, $\sigma_a^2$ =1; 1000 replications )**

| ω | Estimated Value of AIC | | Estimated Value of BIC | |
|---|---|---|---|---|
| | AO | IO | AO | IO |
| 0 | 283.1458 | 283.1458 | 0.1216 | 0.1216 |
| 1 | 285.1909 | 285.1142 | 0.1595 | 0.1587 |
| 2 | 289.1839 | 289.0033 | 0.1806 | 0.1785 |
| 3 | 293.5158 | 292.6654 | 0.2247 | 0.2160 |
| 4 | 303.5555 | 302.4034 | 0.2343 | 0.3129 |
| 5 | 309.9643 | 308.2291 | 0.3880 | 0.3702 |
| 6 | 317.5166 | 315.4541 | 0.4631 | 0.4434 |
| 7 | 325.4445 | 323.1071 | 0.5429 | 0.5193 |
| 8 | 334.7490 | 332.4016 | 0.6356 | 0.6115 |
| 9 | 347.7773 | 344.8703 | 0.7651 | 0.7352 |
| 10 | 356.3748 | 353.8092 | 0.8502 | 0.8244 |

The table below represents the relative changes (RC) for estimated values of AIC and BIC for the generated MA(1) outlier contaminated series with an AO and an IO at t = 50 for ω = 0, 1, . . . , 10, μ = 0, $\sigma_a^2$ = 1 and θ = –0.3.

**Table (3.22)**

**Relative Changes (%) for Average of Estimated Values of AIC and BIC: MA(1) with an AO and an IO at t = 50 (n=100, θ = –0.3, μ = 0, $\sigma_a^2$ =1; 1000 replications)**

| ω | RC for Estimated Value of AIC | | RC for Estimated Value of BIC | |
|---|---|---|---|---|
| | AO | IO | AO | IO |
| 1 | 0.7223 | 0.6952 | 31.1678 | 30.5099 |
| 2 | 2.1325 | 2.0687 | 48.5197 | 46.7928 |
| 3 | 3.6624 | 3.3621 | 84.8762 | 77.6316 |
| 4 | 7.2082 | 6.8013 | 92.6809 | 157.3191 |
| 5 | 9.4716 | 8.8588 | 219.0789 | 204.4408 |
| 6 | 12.1389 | 11.4105 | 280.8388 | 264.6382 |
| 7 | 14.9388 | 14.1133 | 346.4638 | 327.0559 |
| 8 | 18.2250 | 17.3959 | 422.6974 | 402.8783 |
| 9 | 22.8262 | 21.7995 | 529.1941 | 504.6053 |
| 10 | 25.8626 | 24.9565 | 599.1776 | 577.9605 |

According to Table (3.21), it is found that the average of estimated values of AIC and BIC for the generated MA(1) outlier free series ($\omega = 0$) are 283.1458 and 0.1216 respectively. From Tables (3.21) and (3.22), it is observed that the effect of outlier on the AIC and BIC is also much more significant in the case of AO than that of IO for MA(1) series as in the case of AR(1). It is also seen that the increasing values of AIC and BIC are obtained as the values of outlier parameter $\omega$ become larger.

The values of AIC and BIC were also estimated for each of the generated outlier free ($\omega = 0$) ARMA (1, 1) series together their corresponding outlier contaminated series with AO and IO outlier.

The following table represents the average of estimated values of AIC and BIC for the ARMA(1,1) outlier free ($\omega = 0$) series and the corresponding outlier contaminated series with an AO and an IO at $t = 50$ for $\omega = 0, 1, \ldots, 10$, $\mu = 0$, $\sigma_a^2 = 1$ $\phi = 0.5$ and $\theta = -0.3$, based on 1000 replications.

**Table (3.23)**

**Average of Estimated Values of AIC and BIC: ARMA(1,1) with an AO and an IO at t =50 (n=100, $\phi = 0.5, \theta = -0.3, \mu = 0, \sigma_a^2 = 1$; 1000 replications)**

| $\omega$ | Estimated Value of AIC | | Estimated Value of BIC | |
|---|---|---|---|---|
| | AO | IO | AO | IO |
| 0 | 281.8826 | 281.8826 | 0.1167 | 0.1167 |
| 1 | 283.1576 | 282.5657 | 0.1291 | 0.1230 |
| 2 | 287.7783 | 285.2520 | 0.1749 | 0.1491 |
| 3 | 295.8767 | 290.3409 | 0.2577 | 0.2073 |
| 4 | 304.1748 | 296.0799 | 0.3419 | 0.2603 |
| 5 | 314.3469 | 304.1226 | 0.4445 | 0.3414 |
| 6 | 324.8468 | 312.4259 | 0.5495 | 0.4234 |
| 7 | 335.0720 | 321.7155 | 0.6529 | 0.5178 |
| 8 | 345.6052 | 331.3625 | 0.7587 | 0.6143 |
| 9 | 355.8164 | 341.2210 | 0.8608 | 0.7136 |
| 10 | 366.1972 | 350.4690 | 0.9395 | 0.8060 |

The table below represents the relative changes (RC) for estimated values of AIC and BIC for the generated ARMA(1,1) outlier contaminated series with an AO and an IO at $t = 50$ for $\omega = 0, 1, \ldots, 10$, $\mu = 0$, $\sigma_a^2 = 1$, $\phi = 0.5$ and $\theta = -0.3$.

**Table (3.24)**

**Relative Changes (%) for Average of Estimated Values of AIC and BIC: ARMA(1,1) with an AO and an IO at t = 50**

**(n=100, $\phi = 0.5$, $\theta = -0.3$, $\mu = 0$, $\sigma_a^2=1$; 1000 replications)**

| ω | RC for Estimated Value of AIC | | RC for Estimated Value of BIC | |
|---|---|---|---|---|
| | AO | IO | AO | IO |
| 1 | 0.4523 | 0.2423 | 10.6255 | 5.3985 |
| 2 | 2.0915 | 1.1953 | 49.8715 | 27.7635 |
| 3 | 4.9643 | 3.0006 | 120.8226 | 77.6350 |
| 4 | 7.9083 | 5.0366 | 192.9734 | 123.0506 |
| 5 | 11.5170 | 7.8898 | 280.8912 | 192.5450 |
| 6 | 15.2419 | 10.8355 | 370.8655 | 262.8106 |
| 7 | 18.8693 | 14.1310 | 459.4687 | 343.7018 |
| 8 | 22.6061 | 17.5534 | 550.1285 | 426.3925 |
| 9 | 26.2286 | 21.0507 | 637.6178 | 511.4824 |
| 10 | 29.9112 | 24.3315 | 705.0557 | 590.6598 |

According to Table (3.23), it is noticed that the average of estimated values of AIC and BIC for the generated ARMA(1,1) outlier free series (ω = 0) are 281.8826 and 0.1167 respectively. From Tables (3.23) and (3.24), it is observed that the effect of outlier on the AIC and BIC is also much more significant in the case of AO than that of IO for ARMA(1,1) series as in the case of both AR(1) and MA(1) series. It is also seen that the increasing values of AIC and BIC are obtained as the values of outlier parameter ω become larger.

According to the findings presented in this chapter, it is clear that both AO and IO types of outlier affect on the mean ,variance, ACF, PACF, estimated error variance, parameter estimates, AIC and BIC values. Therefore, it can be concluded that there can be the problems in the model identification due to the effects of the outliers.

## 3.6 Effects in Model Identification

Tolvi (1998) suggested that in time series of short to moderate length, often the presence of a single outlier will result in a true AR model being falsely identified as an MA or ARMA model, and that the identified lag lengths (p and q) will also be wrong. Similarly, outliers bias estimated ARMA model parameters. Only AR and MA models are considered in this section because ARMA is a mixed model and it is difficult to decide a model is an ARMA based on the ACF and PACF.

In order to study the effect of outlier in model identification, 100 outlier free ($\omega = 0$) AR (1) series for $n = 100$ and $\phi = 0.7$ were again generated using S-PLUS and an AO and an IO with outlier parameters $\omega = 0, 5, 10$ were introduced at $t = 50$.

In the information criterion approach for model selection, models that yield a minimum value for the criterion are to be preferred, and the AIC or BIC values are compared among various model as the basis for the selection of the model. Since the BIC criterion imposes a greater penalty for the number of estimated parameters than does AIC, use of minimum BIC for model selection would always result in a chosen model whose number of parameters is no greater than that chosen under AIC (Box at el., 1994). Hence, the BIC was also used as the model selection criterion in this study.

To fit each generated AR(1) series, some other commonly used models such as AR(2), MA(1), MA(2), ARMA(1,1), ARMA(2,1), ARMA(1,2) and ARMA(2,2) were considered. The BIC values for each fitted models were computed and compared with the BIC value of generated AR(1) model. If the BIC value of the fitted AR(1) model is minimum among others, it is the correct model selection for the generated AR(1) series. The following table represents the percentages of correct and incorrect model selection cases for the generated AR(1) series based on BIC.

**Table (3.25)**

**Correct and Incorrect Percentages of Model Selection Cases: AR(1) with an AO and an IO at $t = 50$ ($n = 100$ and $\phi = 0.7$; 100 replications)**

| | AO | | IO | |
|---|---|---|---|---|
| $\omega$ | Correct % | Incorrect % | Correct % | Incorrect % |
| 0 | 88 | 12 | 88 | 12 |
| 5 | 86 | 14 | 95 | 5 |
| 10 | 76 | 24 | 93 | 7 |

According to Table (3.25), it can be seen that the percentage of correct model selection decreases as the value of outlier parameter increases in the AO case of AR(1) series whereas the percentage of correct model selection does not decrease as the value of outlier parameter increases in the IO case of AR(1) series. It can be said that the outlier affects on model identification in the case of AR(1) with an AO.

Again, 100 outlier free ($\omega = 0$) MA(1) series for $n = 100$ and $\theta = 0.7$ were generated using S-PLUS Software and an AO and an IO with outlier parameters

$\omega$ = 0, 5, 10, were also introduced at t = 50. Then, each generated MA(1) series was also fitted by some commonly used model as mentioned above and BIC values for each fitted model were also computed and compared with the BIC value of the generated MA(1) model. If the BIC values of the fitted possible models mentioned above are greater than the BIC value of the correct fitted model MA(1), the correct model is obtained for the generated MA(1) series. The following table indicates the percentages of correct and incorrect model selection cases for the generated MA(1) series based on BIC.

**Table (3.26)**

**Correct and Incorrect Percentages of Model Selection Cases: MA(1) with an AO and an IO at t = 50 (n = 100 and $\theta$ = 0.7; 100 replications)**

| $\omega$ | AO | | IO | |
|---|---|---|---|---|
| | Correct % | Incorrect % | Correct % | Incorrect % |
| 0 | 91 | 9 | 91 | 9 |
| 5 | 84 | 16 | 83 | 17 |
| 10 | 76 | 24 | 92 | 8 |

From Table (3.26), it is also shown that the percentage of correct model selection cases declines as the value of outlier parameter increases in the AO case of MA(1) series but the percentage of correct model selection does not decline as the value of outlier parameter increases in the IO case of MA(1) series. Here, it can be said that the outlier affects on model identification in the case of MA (1) with an AO.

According to the simulation results as shown in Tables (3.25) and (3.26), it is clear that the effect of AO outlier is serious for both AR(1) and MA(1) models' identification but effect of IO outlier is not as clear as AO.

Each particular generated series for AR(1) and MA(1) were also considered and the models were fitted by using the ACF and PACF.

Here, under the hypothesis of the underlying process is a white noise sequence, the variance of the ACF $\hat{\rho}_k$ can be approximated by

$$V(\hat{\rho}_k) \approx \frac{1}{n}(1 + 2\hat{\rho}_1^2 + \ldots + 2\hat{\rho}_{k-1}^2) \qquad (3.2.18)$$

and the variance of PACF $\hat{\phi}_{kk}$ can be computed as

$$V(\hat{\phi}_{kk}) \approx \frac{1}{n}. \qquad (3.2.19)$$

Hence $\pm 2$ S. E (Standard Error) can be used as critical limits on the ACF and PACF to test the hypothesis of a white noise process.

The following figures show the plots of the ACF and PACF together with their confidence limits for the generated outlier free AR(1) series as well as its outlier contaminated series of AO ($\omega = 10$) and an IO ($\omega = 10$) at $t = 50$.

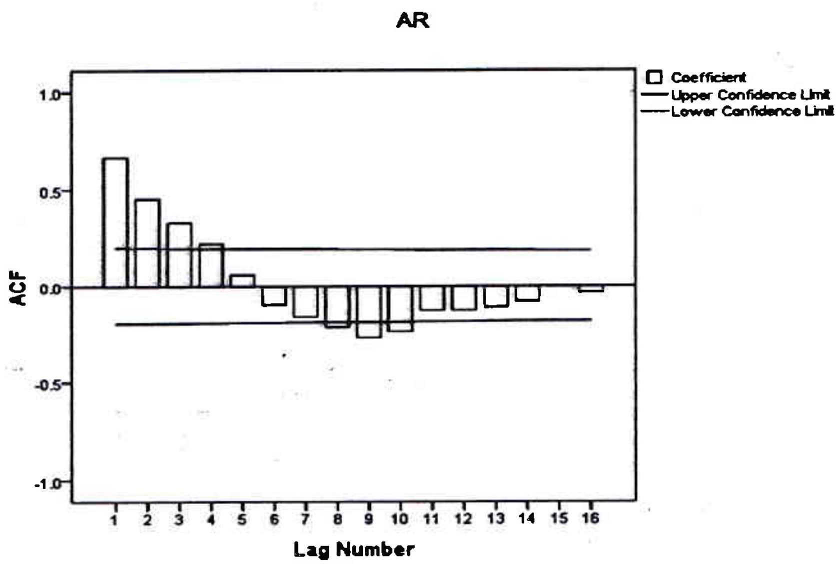


**Figure 3.4 ACF of Generated AR(1) Series ($n = 100, \phi = 0.7$)**

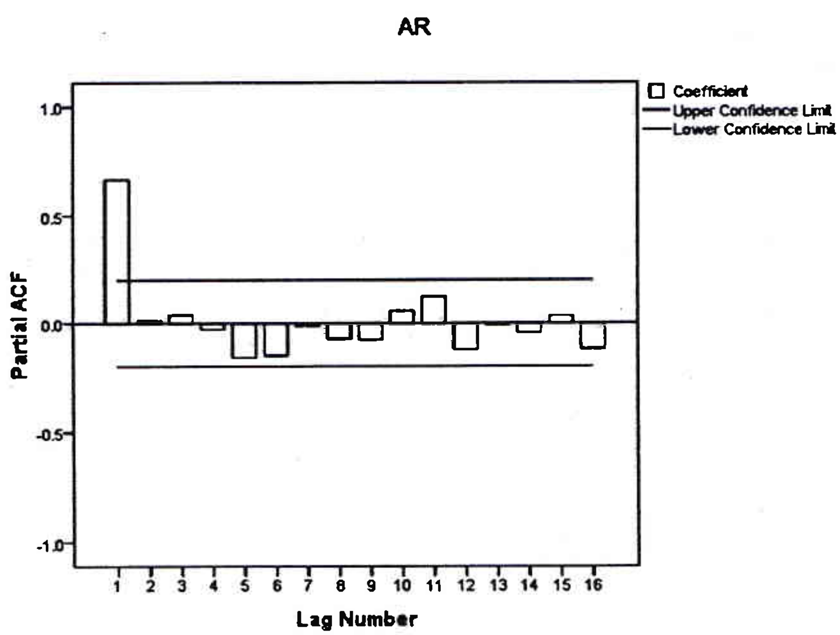


**Figure 3.5 PACF of Generated AR(1) Series ($n = 100, \phi = 0.7$)**

Figures 3.4 and 3.5 show that the generated AR(1) series follows an AR(1) model because the ACF of the generated AR(1) model tails off and the PACF of the model cuts off after lag 1.

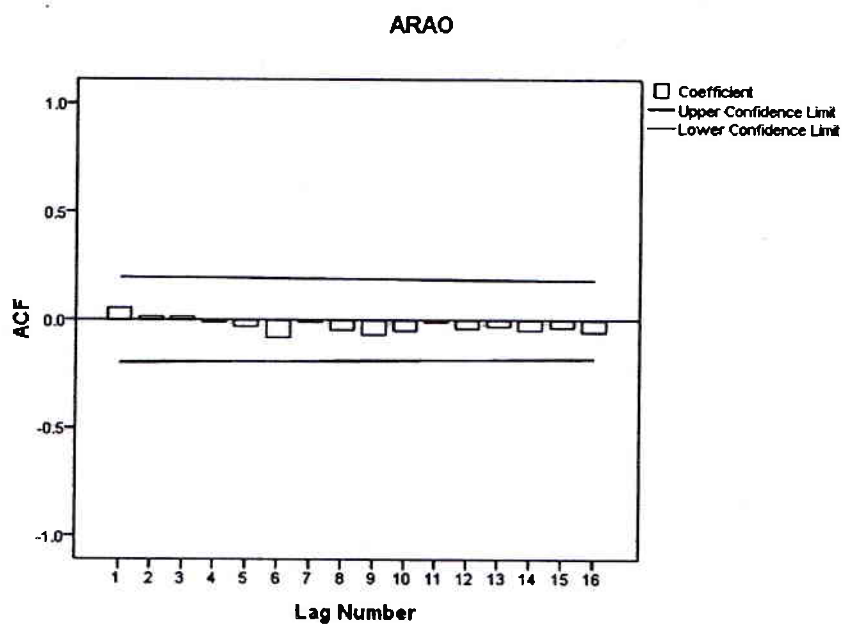


**Figure 3.6** **ACF of Generated AR(1) Series with an AO Outlier at t = 50 (n = 100, $\phi = 0.7$, $\omega = 10$)**

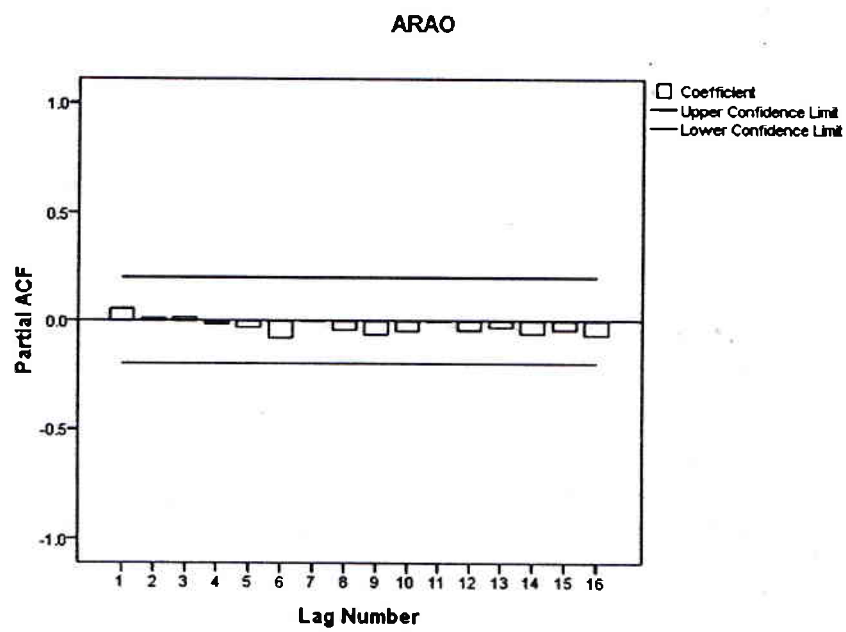


**Figure 3.7** **PACF of Generated AR(1) Series with an AO Outlier at t = 50 (n = 100, $\phi = 0.7$, $\omega = 10$)**

Figures 3.6 and 3.7 indicate that the ACF and PACF of the generated AR(1) series with an AO ($\omega = 10$) at $t = 50$ do not show the theoretical patterns of ACF and PACF for an AR(1) model. It suggests a white noise or random phenomenon. Thus, the generated AR(1) series with an AO outlier at $t = 50$ with $\omega = 10$ does not follow an AR(1)model.

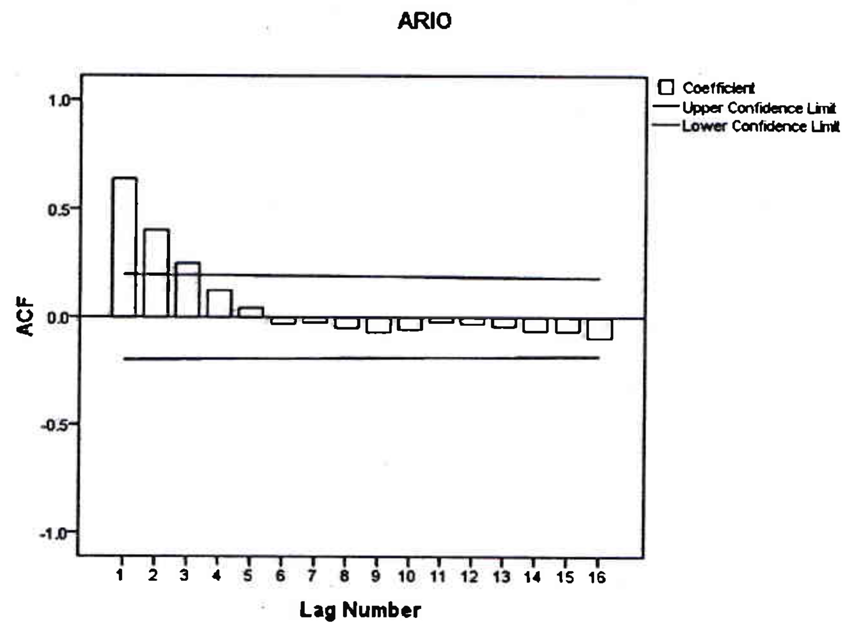


**Figure 3.8** **ACF of Generated AR(1) Series with an IO Outlier at $t = 50$ ($n = 100, \phi = 0.7, \omega = 10$)**

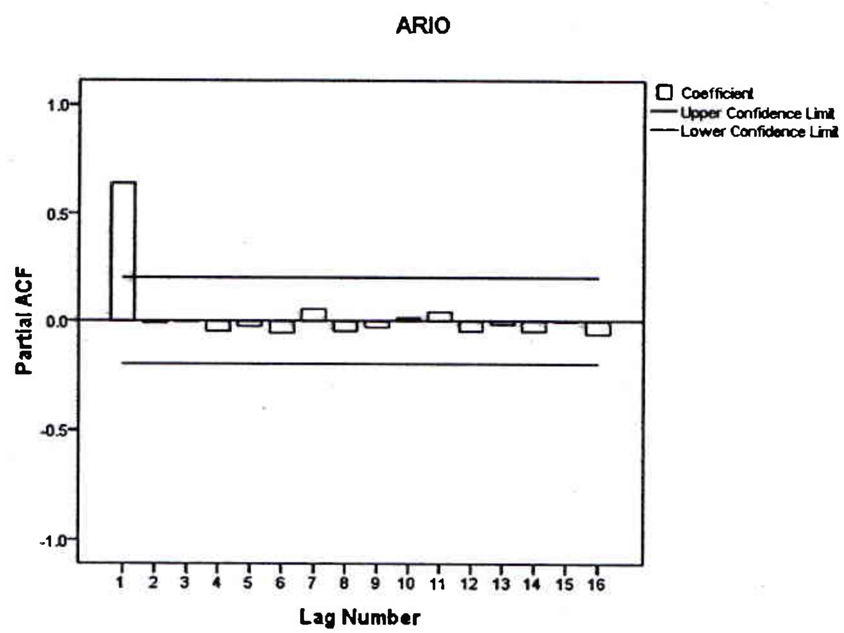


**Figure 3.9** **PACF of Generated AR(1) Series with an IO Outlier at $t = 50$ ($n = 100, \phi = 0.7, \omega = 10$)**

According to Figures 3.8 and 3.9, it is shown that ACF of AR(1) series with an IO outlier (ω = 10) at t = 50 tails off and its PACF cuts off after lag 1. Therefore, AR(1) series with an IO outlier (ω = 10) is identified as an AR(1) model.

The following figures show the plots of the ACF and PACF for the generated outlier free MA(1) series as well as its outlier contaminated series of an AO (ω = 10) and an IO (ω = 10) at t = 50.

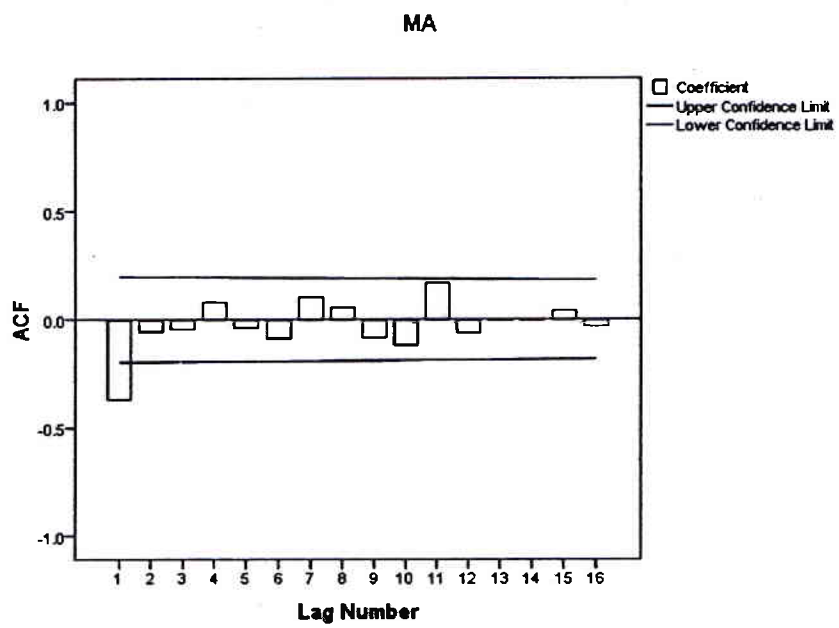


**Figure 3.10 ACF of Generated MA(1) Series (n = 100, θ = 0.7)**

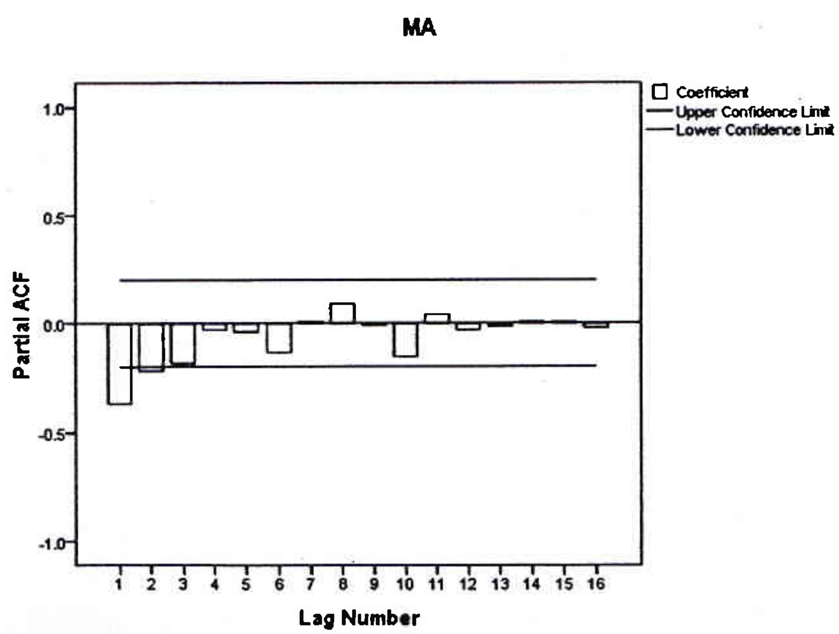


**Figure 3.11 PACF of Generated MA(1) Series (n = 100, θ = 0.7)**

From Figures 3.10 and 3.11, it can be seen that the generated MA(1) series follows MA(1) model because the ACF of the generated series cuts off after lag 1 and the PACF of the model tails off.

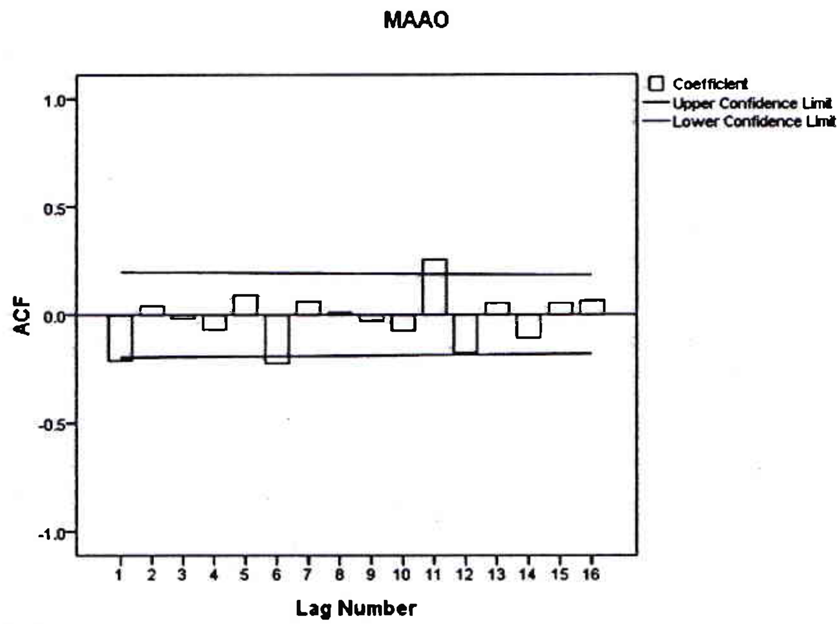


**Figure 3.12 ACF of Generated MA(1) Series with an AO Outlier at t = 50 (n = 100, θ = 0.7, ω = 10)**

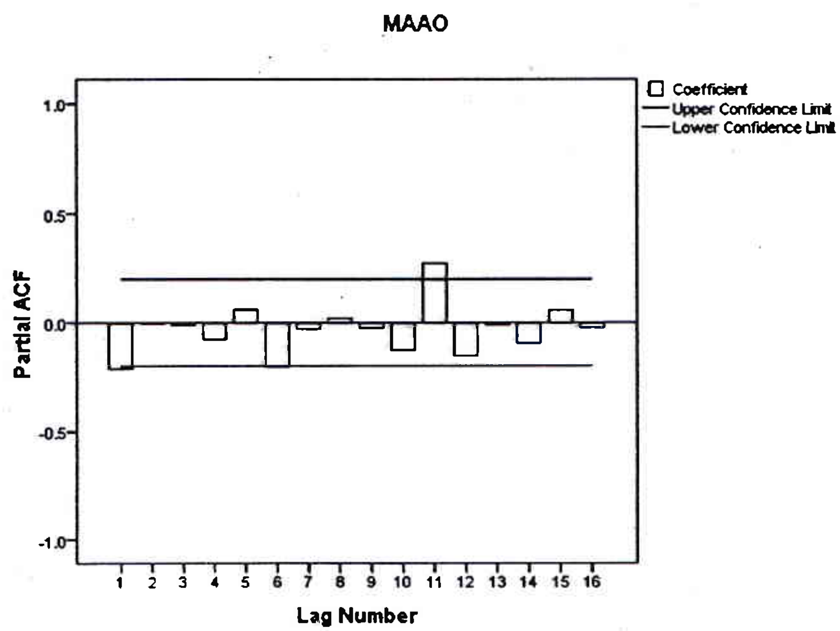


**Figure 3.13 PACF of Generated MA(1) Series with an AO Outlier at t = 50 (n = 100, θ = 0.7, ω = 10)**

Figures 3.12 and 3.13 indicate that the ACF and PACF of the generated MA(1) series with an AO ($\omega$ = 10) at t = 50 do not show the theoretical patterns of ACF and PACF for MA(1) model. Therefore, it can be said that this series does not follow MA(1)model.

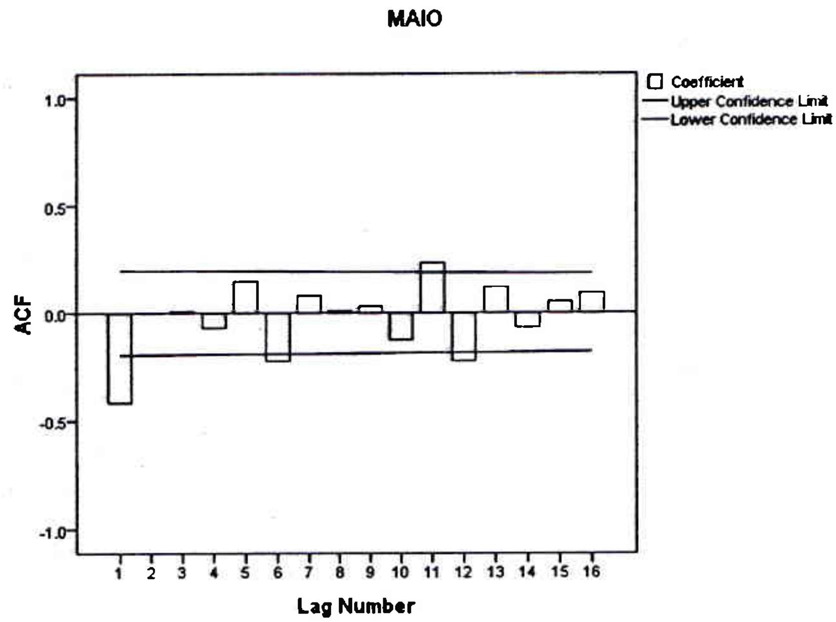


**Figure 3.14 ACF of Generated MA(1) Series with an IO Outlier at t = 50 (n = 100, $\theta$ = 0.7, $\omega$ = 10)**

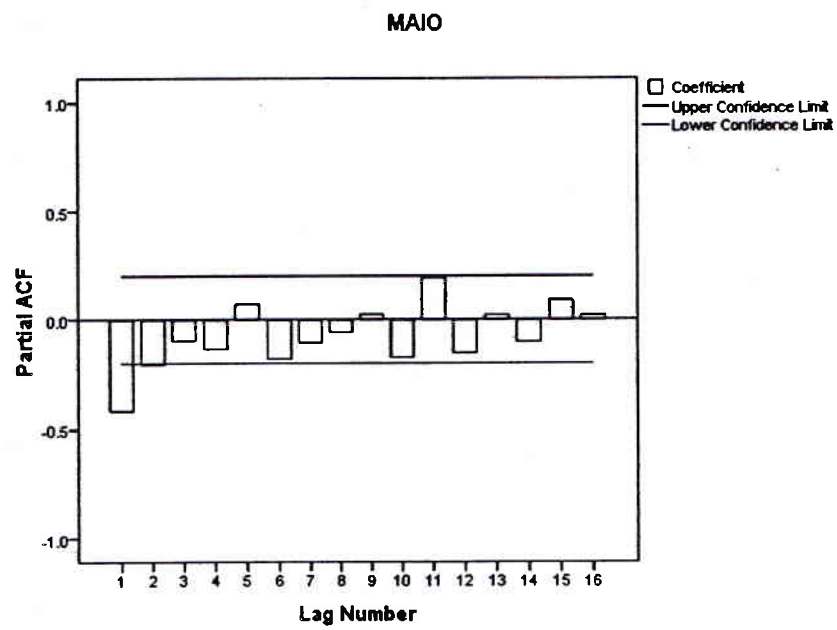


**Figure 3.15 PACF of Generated MA(1) Series with an IO Outlier at t = 50 (n = 100, $\theta$ = 0.7, $\omega$ = 10)**

Figures 3.14 and 3.15 show that ACF of MA(1) series with an IO outlier ($\omega = 10$) at $t = 50$ cuts off after lag 1 and its PACF tails off. Therefore, MA(1) series with an IO outlier ($\omega = 10$) is identified as an MA(1) model.

Based on the simulated results as mentioned above, it can be concluded that the effect of an AO outlier is more serious in model identification but an IO outlier has no effect on model identification for both AR(1) and MA(1) cases.

# CHAPTER IV

# DETECTION AND ESTIMATION OF OUTLIERS IN SELECTED ECONOMIC TIME SERIES

## 4.1 Selected Economic Time Series for Outliers Detection

For detection and estimation of outliers in economic time series, the published figures on Myanmar Trade Statistics and data on Agricultural Production of Myanmar are considered. The data series selected for the detection and estimation of outliers are

1. Base metals and ores export series (from 1955-56 to 2005-06)
2. Teak export series (from 1955-56 to 2005-06)
3. Wheat production series (from 1950-51 to 2005-06)
4. Lablab bean production series (from 1950-51 to 2005-06)
5. Lima bean production series (from 1950-51 to 2005-06)

In this chapter, some ARIMA models with outliers are illustrated for detection and estimation of outliers. The data analysis is carried out using SPSS (Statistical Package for Social Sciences) and STDS (Statistical Time Series Diagnostic Software) for the application of Likelihood Ratio Test (LRT) procedure and Adjustment Diagnostic based on Variance estimate (ADV) method, respectively.

The results on identified outliers in the selected time series are compared. In order to compare the reduction in the estimate of error variance after identification of outliers, the percentage of reduction rate instead of the reduced estimates are used. Apart from reduction in the estimate of error variance, model parsimony is also considered to be an important criterion of analysis. Liu (2006) stated that an important consideration when modeling time series is the *principle of parsimony*. This principle refers to representing the systematic structure of the series with as few parameters as possible. Essentially, this means simpler representations of a time series process are more desirable than more complex ones if both are adequate. This principle leads to the use of mixed ARMA models, rather than just pure AR or pure MA models. The principle of parsimony will be further appreciated when the common occurrence of outliers in time series is taken into consideration.

Moreover, the diagnostic checking for the model adequacy and the simultaneous estimation of the parameters for the fitted models are also presented in this chapter.

## 4.2 Base Metals and Ores Export Series

In this section, "Base Metals and Ores Export Series (in thousand metric ton) from 1955-56 to 2005-06" have been firstly selected for the detection of outliers. The plot of the data series is shown in Figure 4.1.

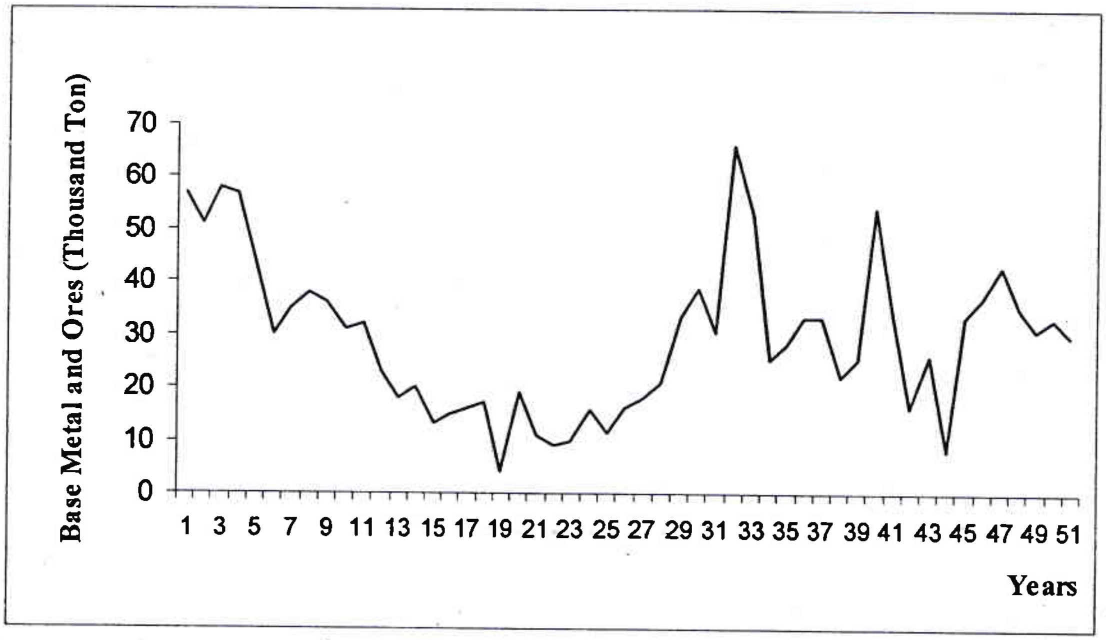


**Figure 4.1 Plot of Base Metal and Ores Export Series (1955-1956 to 2005-2006)**

In Figure 4.1, the unusual peaks at $t = 32, 40$ and troughs at $t = 19, 44$ are found. They look like the possible outliers in this series. But the question is whether all are the outliers or not and what are the types of outliers. It may be difficult to get the correct answer by visualization from the time series sequence plot as in Figure 4.1 and the detection of outliers is an important issue in such case.

Firstly, the identification of the time series model is needed for the observed data series. Using SPSS software, the plots of autocorrelation function (ACF) and partial autocorrelation function (PACF) of the observed series are obtained which show a tail off pattern of ACF and a cut off after lag 1 for PACF respectively. Hence, the model suggested for this data is AR(1) specified by

$$(1 - \phi B)Z_t = \theta_0 + a_t \qquad (4.2.1)$$

where $\theta_0$ is the overall constant in the model. Based on the tentative model AR(1) without outlier , the fitted model is

$$(1 - 0.691 \text{ B})\, Z_t = 30.373 + a_t \qquad (4.2.2)$$

with $\hat{\sigma}_a^2$ = 124.635. Under the model (4.2.1), the likelihood ratio test (LRT) using SPSS and ADV procedure using STDS software are applied to detect the outliers.

Under AR(1) model, the analysis obtained by SPSS shows four outliers at time points t = 32, 34, 40 and 44 by the LRT method with the critical value 3.5 along with the conditional least squares estimation method. The suggested model is

$$Z_t = \theta_0 + \frac{1}{1-\phi B}\left[\omega_1 P_t^{(32)} + \omega_2 P_t^{(34)} + a_t\right] + \omega_3 P_t^{(40)} + \omega_4 P_t^{(44)} \qquad (4.2.3)$$

Before carrying out the adjustment diagnostic on the series, the residual plot (Figure 4.2) and the ADV plot (Figure 4.3) are obtined. The residual plot shows unusual jumps at time points t = 32, 40 and the valleys at t = 34, 44 leading to the suspicion of presence of multiple outliers in the data.

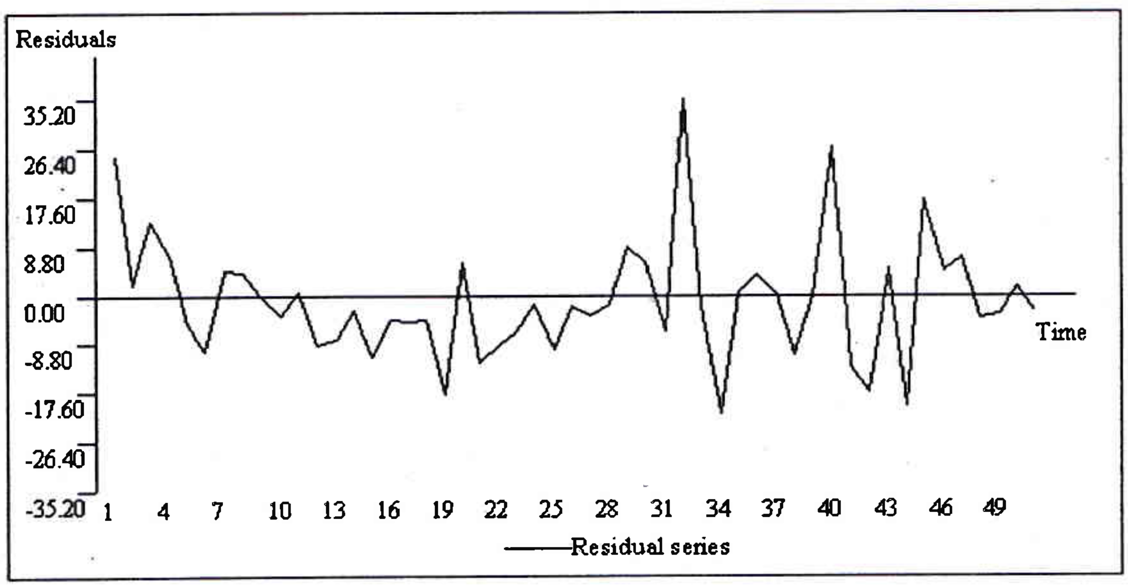


**Figure 4.2 Plot of Residuals for Base Metal and Ores Export Series**

The ADV plot also gives a guideline on selection of warning value. If the warning value 9 is chosen, it can be seen that the ADV plot in Figure 4.3 shows clear peaks, at points t = 32, 40 and 44 indicating presence of outliers.

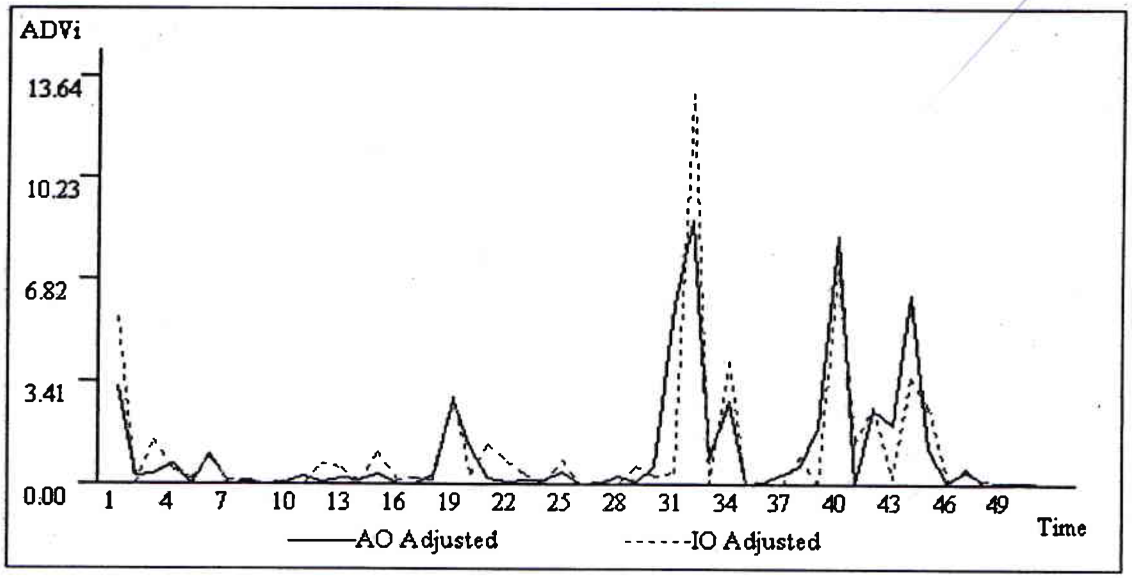


**Figure 4.3 Plot of ADV for Base Metal and Ores Export Series**

The results obtained in carrying out the iterative procedure, and the results of outlier detection analysis by two methods are summarized in the following table.

**Table (4.1)**

**Outlier Detection of Base Metals and Ores Export Series**

| Iteration | LRT | | ADV | |
|---|---|---|---|---|
| | Position | Type | Position | Type |
| 1 | 32 | IO | 32 | IO |
| 2 | 34 | IO | 40 | AO |
| 3 | 40 | AO | 44 | AO |
| 4 | 44 | AO | - | - |

The ADV procedure diagnoses three outliers at time points t = 32, 40 and 44. Hence the suggested model becomes

$$Z_t = \theta_0 + \frac{1}{1-\phi B}\left[\omega_1 P_t^{(32)} + a_t\right] + \omega_2 P_t^{(40)} + \omega_3 P_t^{(44)} \qquad (4.2.4)$$

For the sake of same estimation method and software, the SPSS software is used for the parameter estimation based on the two suggested models (4.2.3) and (4.2.4), the results are presented in Table (4.2).

**Table (4.2)**

**Estimated Parameters of AR(1) with Outliers for Base Metals and Ores Export Series**

| Parameter | Model (4.2.3) | | Model (4.2.4) | |
|---|---|---|---|---|
| | Estimate | S.E | Estimate | S.E |
| $\theta_0$ | 30.398 | 5.663 | 28.076 | 4.654 |
| $\phi$ | 0.837 | 0.068 | 0.773 | 0.076 |
| $\omega_1$ | 35.851 | 7.142 | 36.360 | 7.892 |
| $\omega_2$ | 24.487 | 5.434 | 24.547 | 6.197 |
| $\omega_3$ | -21.513 | 5.433 | -21.453 | 6.196 |
| $\omega_4$ | -24.320 | 7.293 | - | - |
| $\sigma_a^2$ | 61.184 | - | 72.063 | - |

From the last row of the Table (4.2), though both the estimates $\hat{\sigma}_a^2$ based on two models show reduction in comparison with the estimate obtained on ignoring the outliers, their values differ and model (4.2.3) gives a smaller estimate. The reduction percentages in error variances for base metal and ores export series are shown in Table (4.3).

**Table (4.3)**

**The Reduction Percentage in Error Variance for Base Metal and Ores Export Series**

| Model | Estimated Variance | Reduction Percent |
|---|---|---|
| AR(1) with 4 outliers (4.2.3) | 61.184 (124.6349) | 50.91% |
| AR(1) with 3 outliers (4.2.4) | 72.063 (124.6349) | 42.18% |

Note: The values within the parentheses are the estimated variances $\sigma_a^2$ when outliers are ignored.

From the above table, it is found that the reduction percentage of model (4.2.3) is higher than that of model (4.2.4) and the difference between the reduction percentages for two models is only 8.73%. At the same time, model (4.2.4) gives a more parsimonious model, by detecting lesser number of outliers.

For the diagnostic checking of the fitted models, it is needed to check whether the residuals are white noise or not. Thus, the characteristics of the ACF and PACF of the residual series for the fitted models (4.2.3) and (4.2.4) are illustrated as in Figures 4.4 and 4.5 respectively.

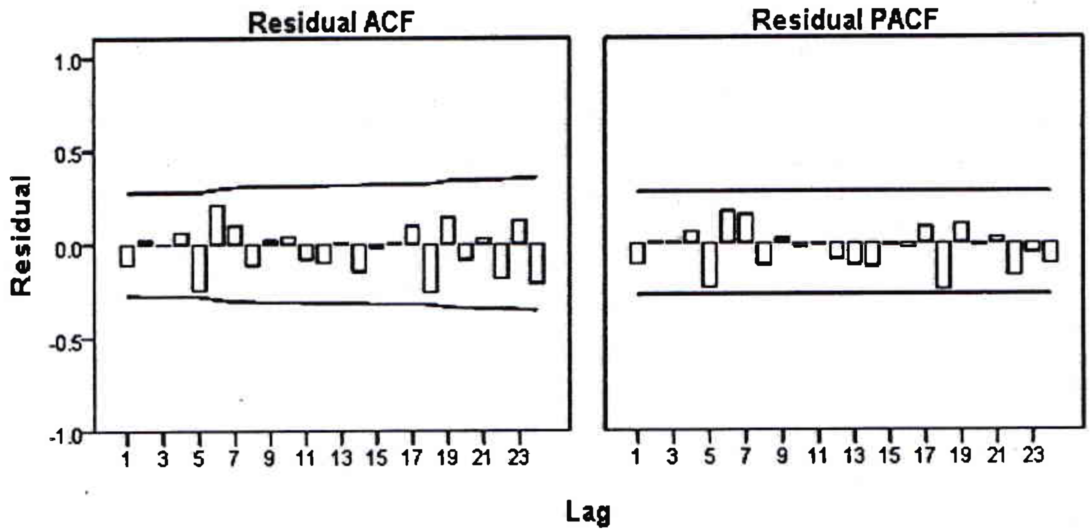


**Figure 4.4 The ACF and PACF of Residual Series for Model (4.2.3)**

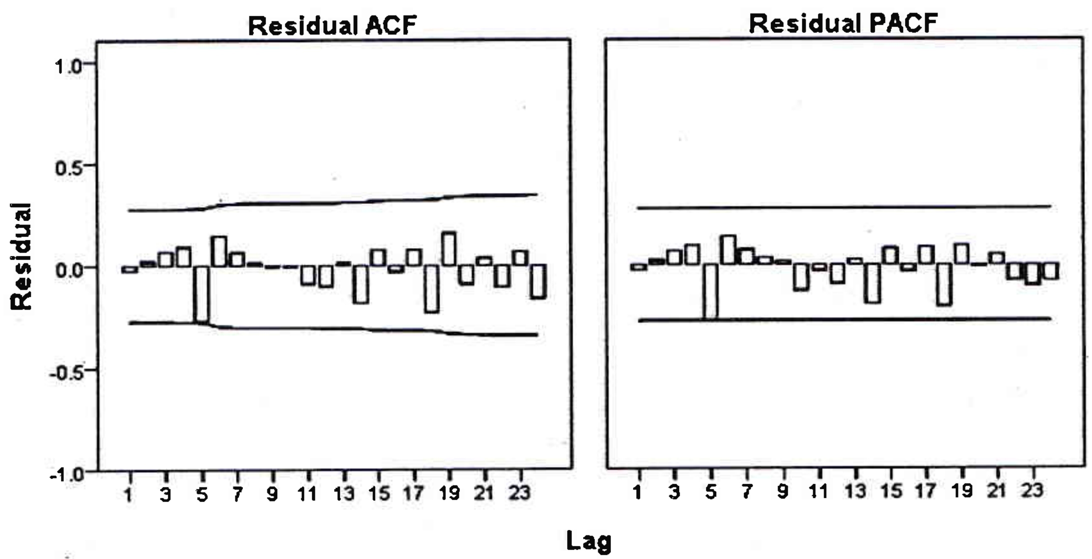


**Figure 4.5 The ACF and PACF of Residual Series for Model (4.2.4)**

From the above figures, it is clear that the residuals of two models do not have any pattern and statistically significant since the estimated ACF of the residual series for both models lie within the confidence limits ($\pm 2$ standard error) at 5% level of significant. This indicates that the population autocorrelations can be taken as zero and the residual series is white noise. Besides, Box-Ljung Q statistics for the two models (4.2.3) and (4.2.4) are obtained as follows.

**Table (4.4)**

**Box-Ljung Q statistic for the Two Models**

| Model | Statistics | DF | Sig. |
|---|---|---|---|
| AR(1) with 4 outliers (4.2.3) | 17.503 | 17 | 0.421 |
| AR(1) with 3 outliers (4.2.4) | 15.319 | 17 | 0.573 |

From Table (4.4), it is found that Box-Ljung Q statistic for the two models are not significant at 5% level of significance which means that the two fitted models are adequate for the base metal and ores export series. Based on the results of these residual analyses, the two tentative models, (4.2.3) and (4.2.4) are adequate for the observed data series.

But, model parsimony was also considered to be an important criterion of analysis. Thus, we can conclude that the model (4.2.4) is more suitable by detecting lesser number of outliers for the base metal and ores export series. Hence, the simultaneous estimation of the parameters of the model is given by

$$Z_t = \underset{(4.654)}{28.076} + \frac{1}{1-\underset{(0.076)}{0.773}B}\left[\underset{(7.892)}{36.360}P_t^{(32)} + a_t\right] + \underset{(6.197)}{24.547}P_t^{(40)} - \underset{(6.196)}{21.453}P_t^{(44)} \qquad (4.2.5)$$

with $\sigma_a^2 = 72.063$ where the values in parentheses below the parameter estimates are the associated standard errors. As we have mentioned above, the fitted model (4.2.5) with an IO at t = 32 (1986-87) and two AO's at t = 40 (1994-95) and t = 44 (1997-98) give us the smaller estimated variance than outlier free model (4.2.1). Moreover, the outliers have an effect on the autoregressive parameter, increasing from 0.691 to 0.773. The effects of outliers on the estimates of model parameters, the model specification, forecasting and impact of outliers in time series modeling are serious problemsin time series analysis. Hence, one should first examine whether there are outliers in the data set before analyzing the time series and should also emphasis on explaining the possible reasons for the occurances of detected outliers in the series.

For base metal and ores export series, a plot of outliers detected can be seen in Figure 4.1. It is of interest to relate these features of outliers to some events which have occurred during the observed period. First, an IO with positive sign at t = 32 indicates that base metal and ores export had risen remarkably up to 66 (thousand metric tons) in 1986-87. It was due to the fact that pure copper, one of the new items of mineral sector, could be produced and exported during this year. In 1987-88, base metal and ores export continued to rise but it dropped afterwards.

Again, an AO with positive sign at t = 40 is detected in 1994-95 when the base metal and ores export rose up to 54 (thousand metric tons) and it was also obvious that more base metal and ores could be produced, as a result of the fact that the State owned corporations made an effort by themselves and they also made concerted effort with both internal and external enterprises.

Finally, an AO with negative sign at t = 44 indicates that base metal and ores export dropped sharply to 8 (thousand metric tons) in 1998-99 as production of base metal and ores fell dramatically during this year. It was due to the fact that the mineral mines became scare, prices of some mineral dropped in world market and consequently production of base metals and ores decreased.

## 4.3 Teak Export Series

The second selected economic time series for the detection of outliers is the "Teak Export Series (in thousand cubic ton) from 1955-56 to 2005-06". The data are plotted in Figure 4.6.

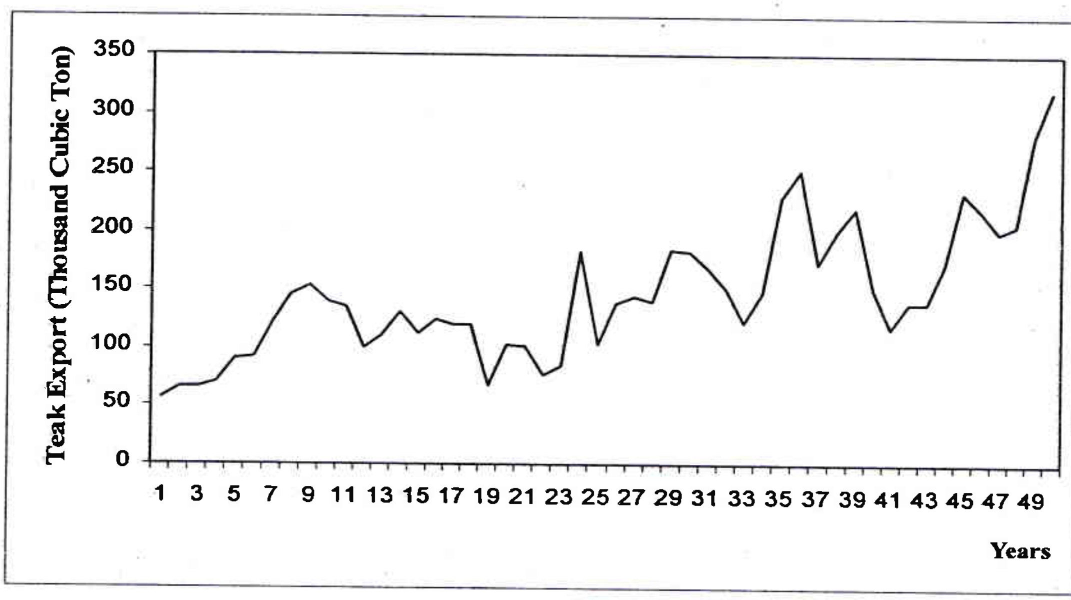


**Figure 4.6 Plot of Teak Export Series (1955-1956 to 2005-2006)**

In the above figure, the values at t = 24, 36, and 46 look like the possible outliers.

For this series, an AR(1) model is also suggested, which is given by

$$(1 - \phi B)Z_t = \theta_0 + a_t$$

and the fitted model with no outlier is obtained as

$$(1 - 0.891 B)Z_t = 160.465 + a_t \quad (4.3.1)$$

with $\hat{\sigma}_a^2 = 1478.864$.

By likelihood ratio test (LRT) under the AR(1) model, AO at t = 24 is identified. Using ADV method by STDS software, an AO is also identified at the position of t = 24. Therefore, the detection of the position and type outliers by the two procedures are identical.

Since both methods detect the same type of outlier at the same position, the suggested AR(1) model with an AO outlier at t = 24 becomes

$$Z_t = \theta_0 + \omega_1 P_t^{(24)} + \frac{1}{1-\phi B} a_t \quad (4.3.2)$$

The estimates of parameters for model (4.3.2) are presented in Table (4.5).

**Table (4.5)**

**Estimated Parameters of AR(1) with an AO Outlier for Teak Export Series**

| Parameter | Estimate | S. E |
|---|---|---|
| $\theta_0$ | 163.207 | 63.29 |
| $\phi$ | 0.932 | 0.075 |
| $\omega_1$ | 88.921 | 23.372 |
| $\sigma_a^2$ | 1224.860 | - |

Hence, the simultaneous estimation of the parameters of the model is given by

$$Z_t = \underset{(63.29)}{163.207} + \underset{(23.372)}{88.921} P_t^{(24)} + \frac{1}{\underset{(0.075)}{1-0.932B}} a_t \quad (4.3.3)$$

with $\hat{\sigma}_a^2 = 1224.860$ where the values in parentheses below the parameter estimates are the associated standard errors.

From the above table, it is also noticed that the estimates of error variance $\sigma_a^2$ from model (4.3.3) reduces in comparison with the estimate obtained on ignoring the outliers from model (4.3.1). The reduction percentage in error variance by model (4.3.3) for teak export series is given in Table (4.6).

**Table (4.6)**

**The Reduction Percentage in Error Variance for Teak Export Series**

| Model | Estimated Variance | Reduction Percent |
|---|---|---|
| AR(1) with an outliers (4.3.3) | 1224.860 | 17.18% |
| AR(1) with no outlier (4.3.1) | 1478.864 | - |

From the Table (4.6), it is noticed that the error variance for model (4.3.3) is smaller than that for model (4.3.1) and the reduction percentage in error variance by model (4.3.3) is 17.18% when the effect of an AO at $t = 24$ is taken into account.

For the diagnostic checking, the residuals series of the fitted model with an outlier is examined to check the residuals are white noise or not. Thus, the ACF and PACF of the residual series for the fitted model with an outlier are plotted in Figures 4.7.

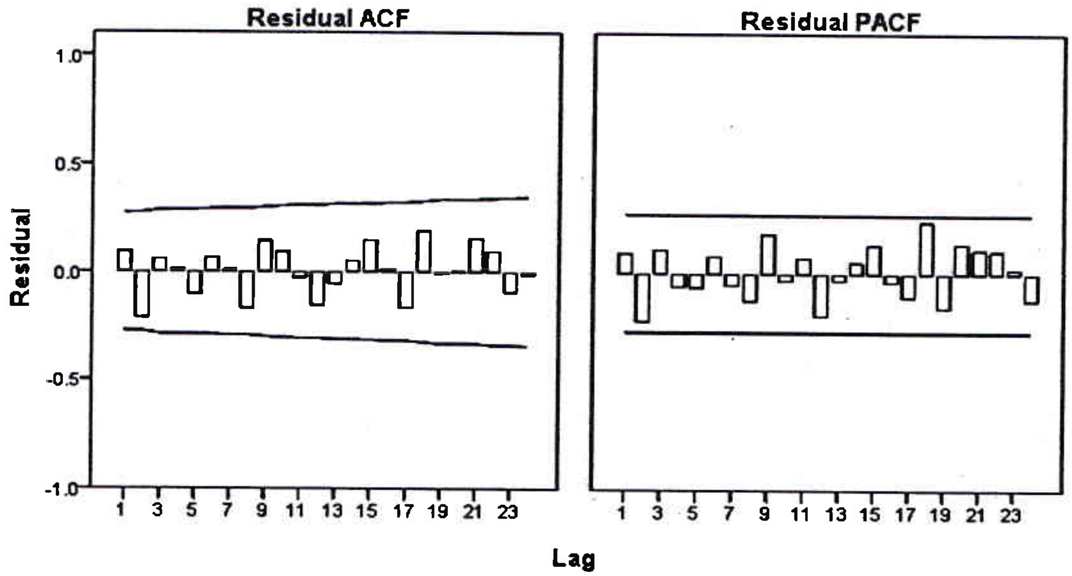


**Figure 4.7 The ACF and PACF of Residual Series for Model (4.3.3)**

From Figure 4.7, the ACF and PACF of the residuals series of the fitted model do not form any pattern and statistically significant since the ACF and PACF lie within two standard deviations for 5% level of significance.

Besides, Box-Ljung Q statistic of the fitted model is 17.028 which is not significant at 5% level of significance. This means that the fitted model is adequate for the teak export series. Based on the results of these residual analyses, the tentative model with an AO at t = 24 (1978-79) is found to be adequate for the observed data series.

First, an AO with positive sign at t = 24 is found when the sharp increase of teak export occurred in 1978-79. It might be due to the fact that as the Timber Extraction Project implemented with World Bank Loan had been completed in 1977-78, the final year of plan period, annual teak production in the ensuring years increased by about 100 thousand cubic tons in 1978-79 and consequently, the export of teak rose to 182.6 thousand cubic tons in this year.

### 4.4 Wheat Production Series

The LRT method and ADV method are now appiled for the detection of outliers in the "Wheat Production Series (in thousand metric ton) from 1950-51 to 2005-06". The data are plotted in the following Figure.

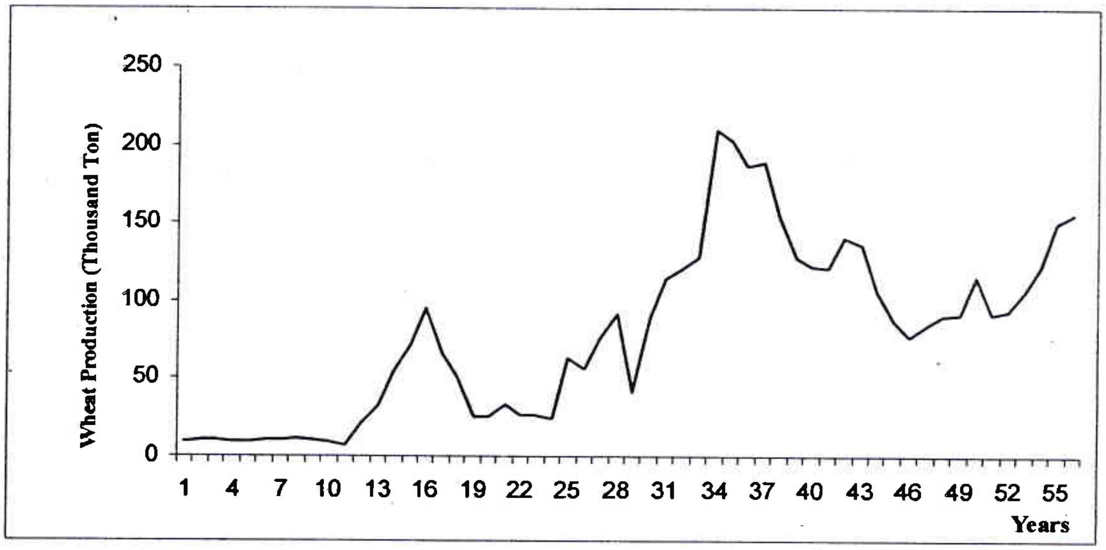


**Figure 4.8 Plot of Wheat Production Series (1950-1951 to 2005-2006)**

In the above figure, the values at t = 16, 29, and 34 look like the possible outliers.

For this series, an AR(1) model is also suggested, which is given by

$$(1 - \phi B)Z_t = \theta_0 + a_t$$

and the fitted model is obtained as

$$(1 - 0.938 B)Z_t = 79.832 + a_t \tag{4.4.1}$$

with $\hat{\sigma}_a^2 = 530.151$.

By likelihood ratio test (LRT) under the AR (1) model, an IO at t = 34 and an AO at t = 29 are identified. Using ADV method, an IO and an AO are also identified at the positions of t = 34 and t = 29 respectively, Therefore, the detection of outliers by the two procedures are identical.

Thus, the fitted AR(1) model with an AO and an IO outlier outliers is given by

$$Z_t = \theta_0 + \frac{1}{1-\phi B}\left[\omega_1 P_t^{(34)} + a_t\right] + \omega_2 P_t^{(29)} \qquad (4.4.2)$$

The estimates of parameters using SPSS for model (4.4.2) are presented in Table (4.7).

**Table (4.7)**

**Estimated Parameters of AR(1) with an AO and an IO Outliers for Wheat Production Series**

| Parameter | Estimate | S. E |
|---|---|---|
| $\theta_0$ | 64.839 | 26.308 |
| $\phi$ | 0.936 | 0.035 |
| $\omega_1$ | 86.220 | 15.628 |
| $\omega_2$ | -49.744 | 11.223 |
| $\sigma_a^2$ | 287.981 | - |

Hence, the simultaneous estimation of the parameters of the model is given by

$$\underset{(26.308)}{Z_t = 64.839} + \frac{1}{\underset{(0.035)}{1-0.936B}}\left[\underset{(15.628)}{86.220} P_t^{(34)} + a_t\right] - \underset{(11.223)}{49.744}\,\omega_2 P_t^{(29)} \qquad (4.4.3)$$

with $\hat{\sigma}_a^2$ = 287.981 where the values in parentheses below the parameter estimates are the associated standard errors.

From the above table, it is also noticed that the estimate of error variance $\sigma_a^2$ from model (4.4.3) reduces in comparison with the estimate obtained on ignoring the outliers from model (4.4.1). The reduction percentage in error variance by model (4.4.3) for wheat production series is given in Table (4.8).

**Table (4.8)**

**The Reduction Percentage in Error Variance for Wheat Production Series**

| Model | Estimated Variance | Reduction Percent |
|---|---|---|
| AR(1) with 2 outliers (4.4.3) | 287.981 | 45.67% |
| AR(1) with no outlier (4.4.1) | 530.151 | - |

From Table (4.8), it is noticed that the error variance for model (4.4.3) is smaller than that for model (4.4.1) and the reduction percentage in error variance by model (4.4.3) is 45.67% when the effects of an IO at t = 34 and an AO at t = 29 are taken into account.

For the diagnostic checking, the residuals series of the fitted model with two outliers is examined to check the residuals are white noise or not. Thus, the ACF and PACF of the residual series for the fitted model with outliers are plotted in Figures 4.11.

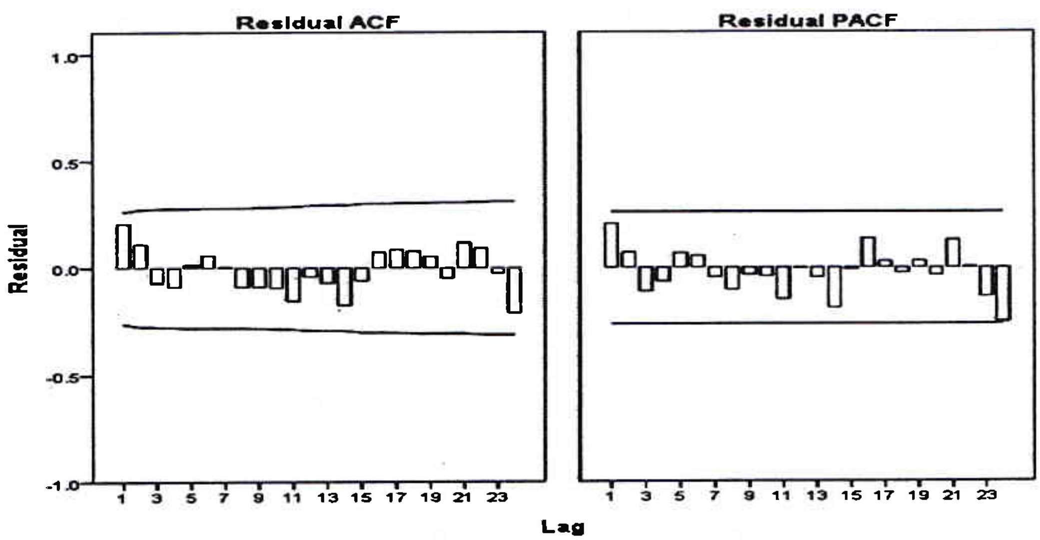


**Figure 4.9 The ACF and PACF of Residual Series for Model (4.4.3)**

From Figure 4.9, the ACF and PACF of the residuals series of the fitted model do not form any pattern and statistically significant since the ACF and PACF lie within two standard deviations for 5% level of significance.

Besides, Box-Ljung Q statistic of the fitted model is 12.407 which is not significant at 5% level of significance. This means that the fitted model is adequate for the wheat production series. Based on the results of these residual analyses, the tentative model with an AO at t = 29 (1978-79) and an IO at t = 34 (1983-84) is found to be adequate for the observed data series.

First, an AO with negative sign at t = 29 is found when the sharp fall of wheat production occurred in 1978-79. It might be due to the fact that the number of irrigated area of wheat cultivation decreased relative to the previous years, and the utilization of fertilizer and amount of pure strain of seeds distributed also declined. Thus, the wheat production declined to 41 thousand metric tons in 1978-79.

Second, an IO with positive sign at t = 34 is detected as wheat production increased to 210.2 thousand metric tons in 1983-84. The causes of such increase were increase in the supply of pure strain of seeds, the utilization of fertilizer and insecticides as wheat was considered as one of the most important crops. Besides, new pure strain seeds, which became suitable for the climate of Myanmar, could be produced after doing necessary agricultural research. Then, the State took necessary measures to supply the pure strain of seeds to the cultivators. And, it rained enough for the crops in late monsoon days, then the volume of wheat production not only surpassėd the production of previous years but also it exceeded the production target. Similarly, production of wheat in 1984-85 continued rising but it dropped considerably in years to come.

## 4.5 Lablab Bean Production Series

The data series consider here is the "Production of Lablab Bean (in thousand metric ton) during 1950-51 and 2005-06". The data series is plotted in Figure 4.10.

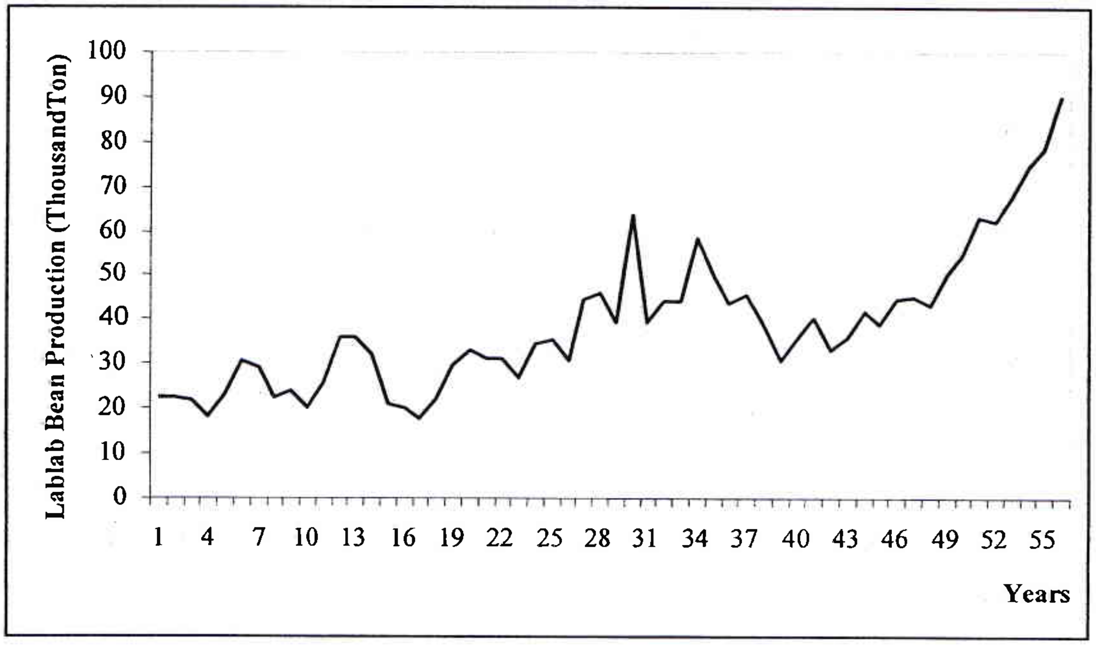


**Figure 4.10 Plot of Lablab Bean Production Series (1950-1951 to 2005-2006)**

In the above figure, the values at t = 30 and t = 34 look like the possible outliers.

For the lablab bean production series an AR(1) model is suggested. The suggested model and fitted models are

$$(1 - \phi B)Z_t = \theta_0 + a_t$$

and

$$(1 - 0.944\ B)Z_t = 45.489 + a_t \qquad (4.5.1)$$

with $\hat{\sigma}_a^2 = 66.765$.

An AO outlier is identified at t = 30 by both LR test and ADV method using STDS. Thus, the detection outliers using both methods are in the same fashion.

Therefore, the fitted AR(1) model with an AO outlier at t = 30 in obtained as

$$Z_t = \theta_0 + \omega_1 P_t^{(30)} + \frac{1}{1-\phi B} a_t \qquad (4.5.2)$$

The estimates of parameters for lablab bean production series using SPSS are presented in Table (4.9).

**Table (4.9)**

**Estimated Parameters of AR(1) with an AO Outlier for Lablab Bean Production Series**

| Parameter | Estimate | S. E |
|---|---|---|
| $\theta_0$ | 48.387 | 37.729 |
| $\phi$ | 0.976 | 0.059 |
| $\omega_1$ | 24.558 | 4.328 |
| $\sigma_a^2$ | 49.546 | - |

Since both detection methods detect same type of outlier at same position, the simultaneous estimation of the parameters of the model is given by

$$Z_t = \underset{(37.729)}{48.387} + \underset{(4.328)}{24.558} P_t^{(30)} + \frac{1}{1-\underset{(0.059)}{0.976}B} a_t \qquad (4.5.3)$$

with $\hat{\sigma}_a^2 = 49.546$ where the values in parentheses below the parameter estimates are the associated standard errors.

From the above table, it is also noticed that the estimates of error variance $\sigma_a^2$ from model (4.5.3) reduces in comparison with the estimate obtained on ignoring the outliers from model (4.5.1).

The reduction percentage in error variance for lablab bean production series is given in Table (4.10).

**Table (4.10)**

**The Reduction Percentage in Error Variance for Lablab Bean Production Series**

| Model | Estimated Variance | Reduction Percent |
|---|---|---|
| AR(1) with an outlier (4.5.3) | 49.546 | 25.79% |
| AR(1) with no outlier (4.5.1) | 66.765 | - |

From the Table (4.10), it is noticed that the error variance for model (4.5.3) is smaller than that for model (4.5.1) and the reduction percentage in error variance by model (4.5.3) is 25.79% when the effects of an AO at t = 30 are taken into account.

For the diagnostic checking, the residuals series of the fitted model with an AO outlier is examined to check the residuals are white noise or not. Thus, the ACF and PACF of the residual series for the fitted model with an outlier are plotted in Figures 4.11.

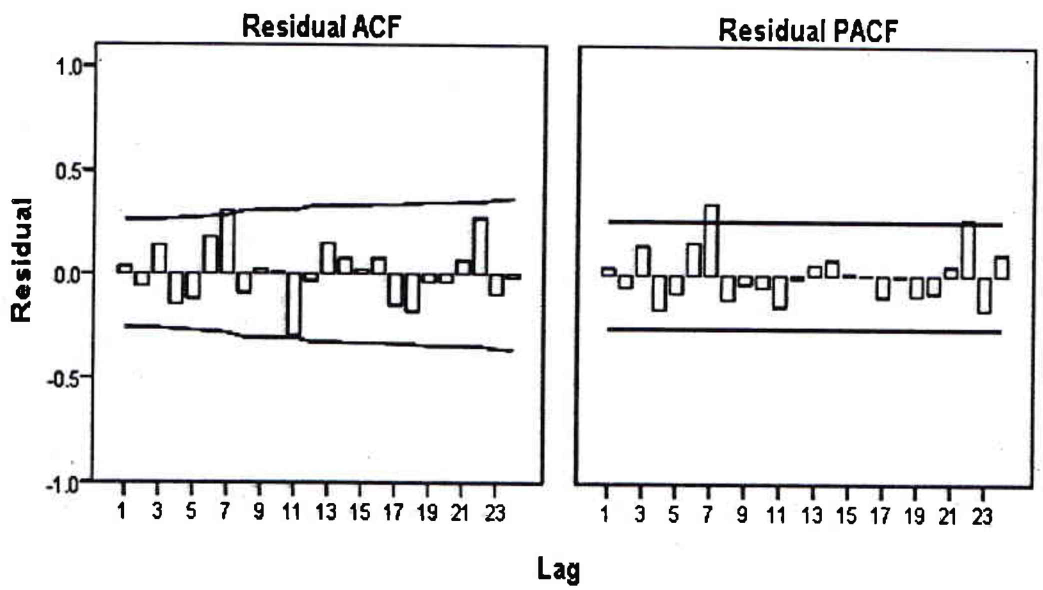


**Figure 4.11 The ACF and PACF of Residual Series for Model (4.5.3)**

From Figure 4.11, the ACF and PACF of the residuals series of the fitted model do not have any pattern and statistically significant since the ACF and PACF lie within two standard deviations for 5% level of significance except for a few lags.

Besides, Box-Ljung Q statistic of 26.562 the fitted model is not significant at 5% level of significance. This means that the fitted model is adequate for the lablab bean production series. Based on the results of these residual analyses, the fitted model (4.5.3) with an AO at t = 30 (1979-80) is found to be adequate for the observed data series.

For lablab bean production series, an AO with positive sign at t = 30 is detected in 1979-80. This is related to lablab bean production obviously increased in 1979-80 with total yield of 63.8 thousand metric tons. It was a result of providing and using of more fertilizers, insecticides and availability of pure strain of seeds in that year. Additionally, varieties of beans had been started to grow in that year, with technical assistance from the UNDP; and leadership of the State, concerted effort of farmers and staff of the departments concerned, and availability of arable area also led to abundant harvest.

## 4.6 Lima Bean Production Series

The data series considered in this section for the detection of outliers is "Lima Bean Production Series (in thousand metric ton) during 1950-51 and 2005-06". The lima bean production series is plotted in Figure 4.12.

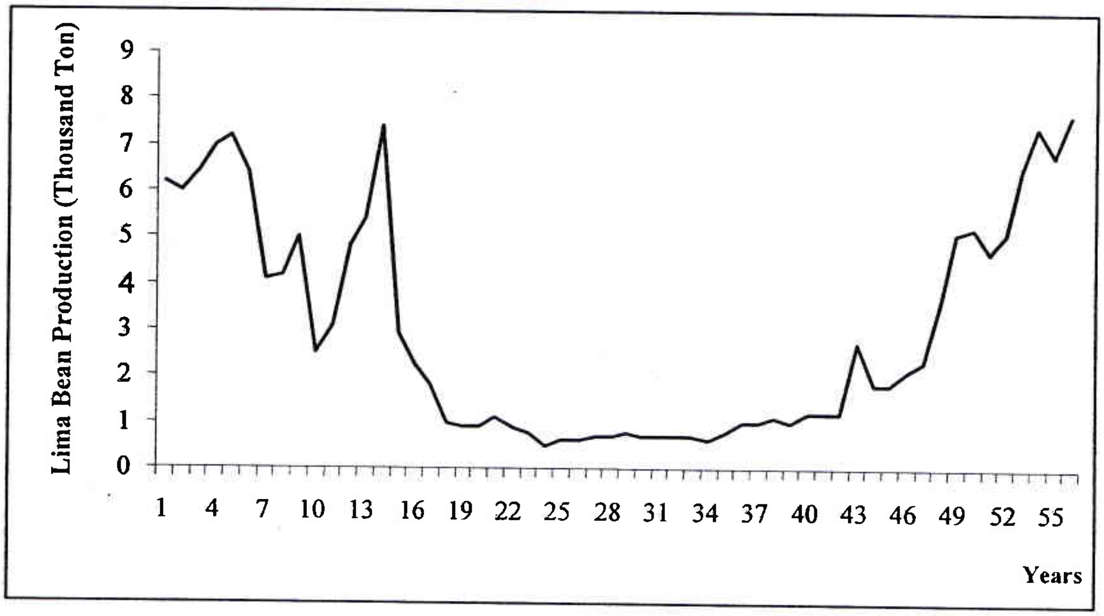


**Figure 4.12 Plot of Lima Bean Production Series (1950-1951 to 2005-2006)**

In the above figure, it is found that the unusual peaks at t = 5, 9, 14, and 43. They seem to be possible outliers.

Using SPSS software, the AR(1) model

$$(1 - \phi B)Z_t = \theta_0 + a_t$$

is suggested for this data series. The fitted model is given by

$$(1 - 0.934 B)Z_t = 4.31 + a_t \qquad (4.6.1)$$

with the variance $\hat{\sigma}_a^2$ = 1.0486 by the conditional least square estimation procedure.

Under AR(1) model, the analysis obtained by LRT method shows three outliers such as an AO at time point t = 14, two IO's at time points t = 7. The suggested outlier model is

$$Z_t = \theta_0 + \frac{1}{1-\phi B}\left[\omega_1 P_t^{(7)} + \omega_2 P_t^{(10)} + a_t\right] + \omega_3 P_t^{(14)} \qquad (4.6.2)$$

Before carrying out the adjustment diagnostic on the series, the residual plot (Figure 4.13) and the ADV plot (Figure 4.14) are plotted.

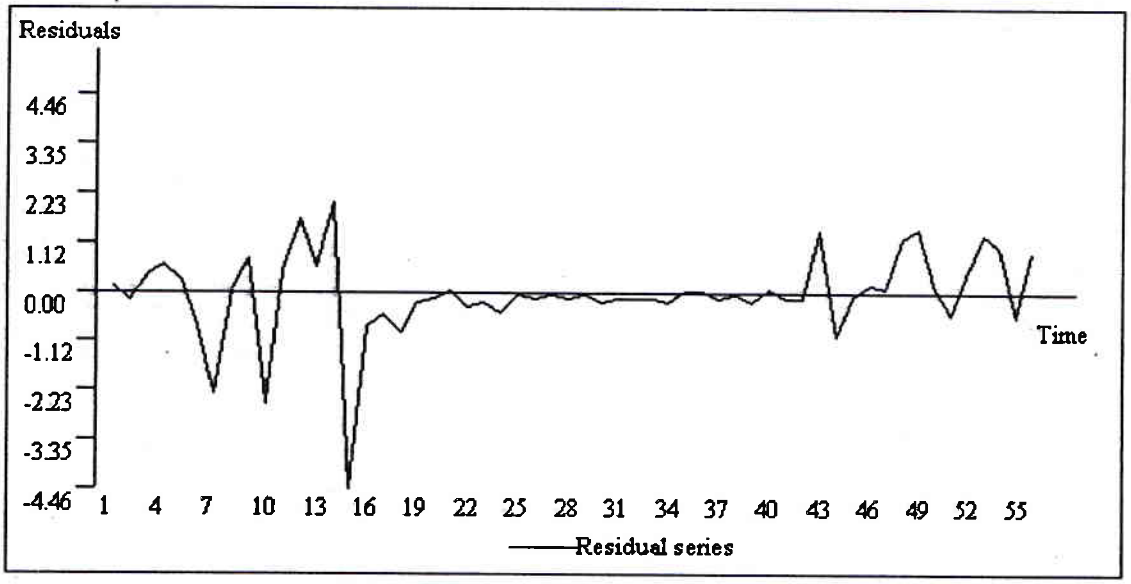


**Figure 4.13 Plot of Residuals for Lima Bean Production Series**

The above residual plot shows unusual points at time t = 14 to the suspicion of presence of outlier in the data.

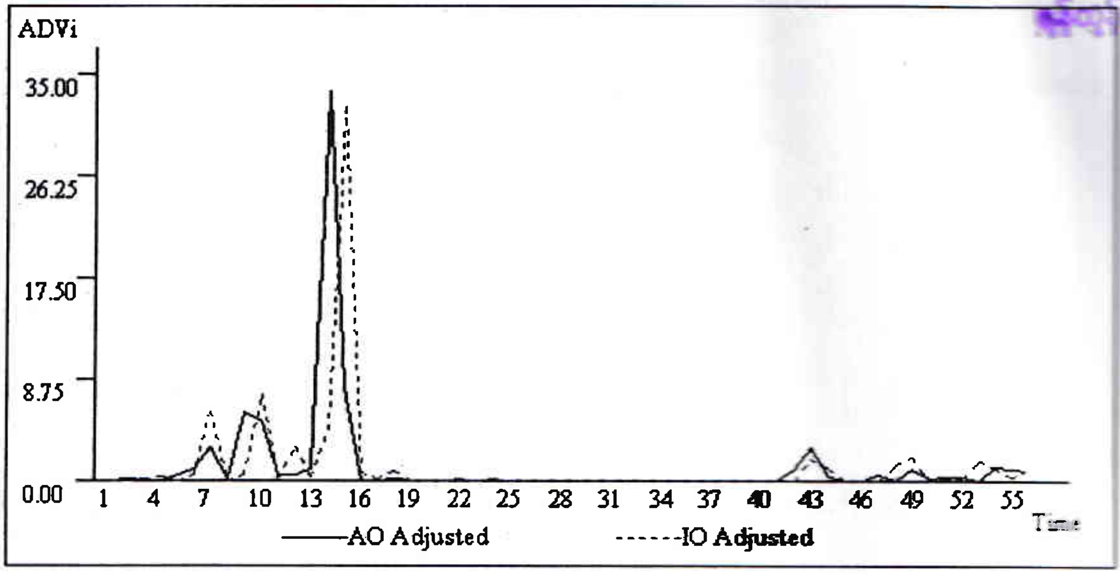


**Figure 4.14 Plot of ADV for Lima Bean Production Series**

The ADV plot is now applied to detect the outliers in lima bean production series. The above ADV plot indicates that the observation at t = 14 is a possible outlier of AO type. Based on the ADV plot, we select the warning value to be 9.

The results obtained on carrying out the iterative procedure, and the results of outlier detection analysis by two methods are summarized in Table (4.11) below.

**Table (4.11)**

**Outlier Detection of Lima Bean Production Series**

| Iteration | LRT | | ADV | |
|---|---|---|---|---|
| | Position | Type | Position | Type |
| 1 | 7 | IO | 14 | AO |
| 2 | 10 | IO | - | - |
| 3 | 14 | AO | - | - |

The ADV procedure diagnoses an AO at t = 14. Hence the suggested model becomes

$$Z_t = \theta_0 + \omega_1 P_t^{(14)} + \frac{1}{1-\phi B} a_t \qquad (4.6.3)$$

For the sake of same estimation method and software, we use the SPSS software for the parameters estimation based on the two suggested models (4.6.2) and (4.6.3), the estimation results for lima bean production series are presented in Table (4.12).

**Table (4.12)**

**Estimated Parameters of AR(1) with Outliers for Lima Bean Production Series**

| Parameter | Model (4.6.2) | | Model (4.6.3) | |
|---|---|---|---|---|
| | Estimate | S.E | Estimate | S.E |
| $\theta_0$ | 6.672 | 2.775 | 4.813 | 2.096 |
| $\phi$ | 0.974 | 0.025 | 0.961 | 0.037 |
| $\omega_1$ | -2.307 | 0.651 | 3.249 | 0.570 |
| $\omega_2$ | -2.543 | 0.650 | - | - |
| $\omega_3$ | 3.249 | 0.464 | - | - |
| $\sigma_a^2$ | 0.423 | - | 0.658 | - |

From the last row of the Table (4.12), though both the estimates $\sigma_a^2$ based on the two models show reduction in comparison with the estimate obtained on ignoring the outliers, their values differ and model (4.6.2) gives a smaller estimate of error variance. The reduction percentages in error variances for lima bean production series are shown in the following table.

**Table (4.13)**

**The Reduction Percentage in Error Variance for Lima Bean Production Series**

| Model | Estimated Variance | Reduction Percent |
|---|---|---|
| AR(1) with 3 outliers (4.6.2) | 0.423 (1.049) | 59.77% |
| AR(1) with 1 outlier (4.6.3) | 0.658 (1.049) | 37.30% |

Note: The values within the parentheses are the estimated variances $\sigma_a^2$ when outliers are ignored.

In the above table, the reduction percentages in error variances obtained from model (4.6.2) is higher than those obtained from model (4.6.3) and the difference between the reduction percentages is 22.47%. At the same time, by detecting less number of outliers model (4.6.3) gives a more parsimonious model.

For the diagnostic checking of the fitted models, it is needed to check whether the residuals are white noise or not. Thus, the characteristics of the ACF and PACF of the residual series for the fitted models (4.6.2) and (4.6.3) are illustrated as in Figures 4.15 and 4.16 respectively.

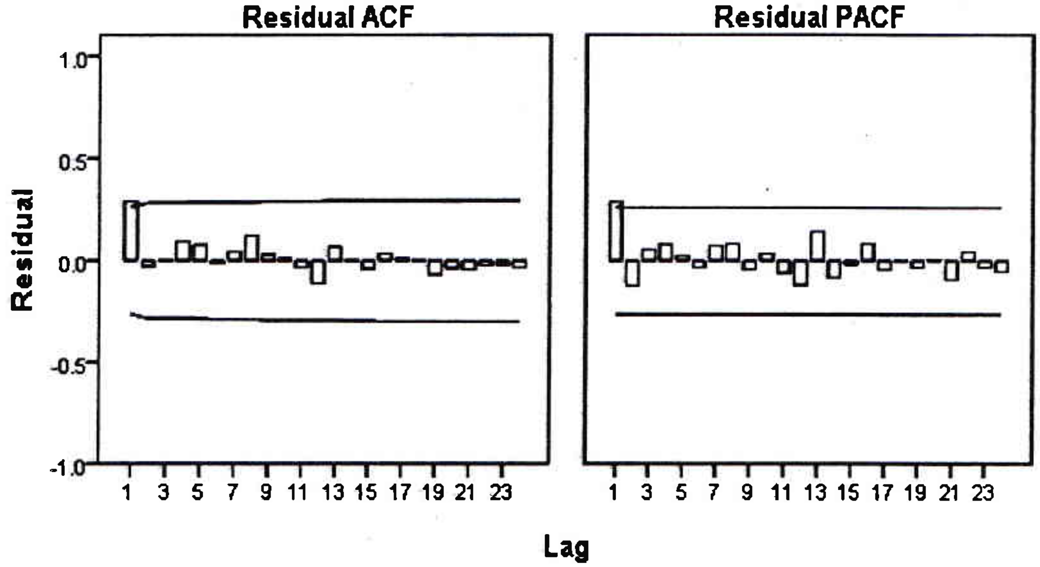


**Figure 4.15 The ACF and PACF of Residual Series for Model (4.6.2)**

From the above figures, it can be seen that the lag 1 ACF and PACF of the residuals of model (4.6.2) do not lie within the confidence limits ($\pm 2$ standard error) at 5% level of significance. This indicates that the population autocorrelations cannot be taken as zero and the residual series is not white noise. Based on the results of these residual analyses, the model (4.6.2) is not adequate for the lima bean production series.

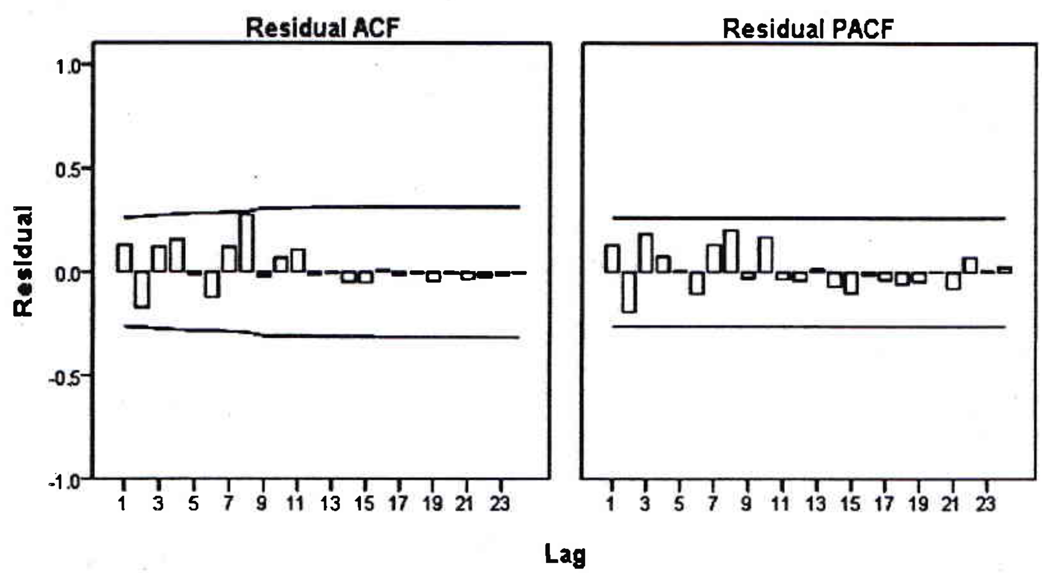


**Figure 4.16 The ACF and PACF of Residual Series for Model (4.6.3)**

From the above figures, it is clear that the residuals of model (4.6.3) do not have any pattern and statistically significant since the estimated ACF of the residual series for model (4.6.3) lie within the confidence limits ($\pm 2$ standard error) at 5% level of significance. This indicates that the population autocorrelations can be taken as zero and the residual series is white noise.

Besides, Box-Ljung Q statistic of the fitted model 15.319 is not significant at 5% level of significance. This means that the fitted model is adequate for the lima bean production series. Based on the results of these residual analyses, the assumed model (4.6.3) with an AO at t = 14 is found to be adequate for the observed data series.

Moreover, model parsimony was also considered to be an important criterion of analysis. Thus, we can conclude that the model (4.6.3) is more suitable by detecting lesser number of outliers for the lima bean production series. Hence, the simultaneous estimation of the parameters of the model is given by

$$\underset{(2.096)}{Z_t = 4.813} + \underset{(0.570)}{3.249P_t^{(14)}} + \frac{1}{\underset{(0.037)}{1 - 0.961B}} a_t \qquad (4.6.4)$$

with $\hat{\sigma}_a^2 = 0.658$ where the values in parentheses below the parameter estimates are the associated standard errors. As we have mentioned above, the fitted model (4.6.4) with an AO at t = 14 (1963-64) gives us the smaller estimated variance than outlier free model (4.6.1). Moreover, the outliers have an effect on the autoregressive parameter, increasing from 0.934 to 0.961.

For lima bean production series, an AO with positive sign at t = 14 is found in 1963-64, as can be seen in Figure 4.12. This pointed out that lima bean production reached up to 7.4 thousand metric tons in 1963-64. The main cause of such a rise was farmer's ability to grow lima bean extensively, due to more demand for that bean; they grew more as the earned favorable price last year. Moreover, vermicelli makers in Monywa District bought them a lot, and as lablab bean crop could not be grown in time in some districts, lima bean were grown in place of lablab bean. Besides, the State encouraged the farmers to grow subsidiary crops, as there was wide availability acreage of alluvial soil along the flooded river banks; they could extend their acreage, and the plants got enough rain in that year, leading to a good harvest.

# CHAPTER V

# FORECASTING AND FORECAST EVALUATION FOR SELECTED ECONOMIC TIME SERIES WITH OUTLIERS

## 5.1 Forecasting and Forecast Evaluation

Forecasting is most often part of a larger process of planning and managing. It is necessary to provide accurate estimates of the future for this larger process. A forecast is a probabilistic estimate or description of a future value or condition. The forecasting is clearly an important factor in planning and decision making. It also plays a crucial role in business, industry, government and institutional planning because many important decisions depend on the anticipated future values of certain variables.

The most important aspect in the conduct of time series analysis is the use of past and present data or available observations to predict future values. Usually, a time series can be exactly predicted in which case it is considered to be deterministic. However, most of the time series data encountered in real situations are stochastic in nature in that the future values are only partially determined by past values. In this case, exact predictions are impossible. Instead, future values are logically thought of as having some probability distribution, which is based upon past values. The use of available observations at time "t" to predict or forecast at some future time "t+L" can serve many purposes of economic and business planning. As Nelson (1973) stated "The better the forecasts available to management are, the better their performance as measured by the outcomes of decision will be."

Forecasts for the time series data can be formed in various ways. The method chosen depends on the purpose. Box and Jenkins (1976) demonstrated that forecasts from ARIMA models are said to be optimal forecasts. This means that no other univariate forecasts have a smaller mean-squared forecast error (MSE).

In this chapter, the minimum mean square error forecast for the fitted models presented in Chapter IV are obtained together with their respective 95% lower and upper confidence limits. These models can also be used to update forecasts when new information becomes available. Moreover, the mean absolute percentage error (MAPE) values are also computed for each of the fitted models to check accuracy of the forecasts.

## 5.2 Forecast Evaluation Measures

A model's forecast accuracy can be enhanced by measuring both absolute and relative measures of error. But, the absolute measures such as mean of error (ME), mean absolute deviation (MAD), sum of squared errors (SSE), mean squared errors (MSE) and residual standard error (RSE) are very much dependent on the scale of the dependent variable. Also, these absolute measures do not allow comparisons of results over time or between time series. Fortunately, there are several relative measures of forecast accuracy that facilitate model comparisons, including percentage error (PE), mean percentage error (MPE) and mean absolute percentage error (MAPE) which can be computed using the following formulas

$$PE = \frac{e_t}{A_t} \times 100$$

$$MPE = \frac{1}{n}\sum_{t=1}^{n}\left(\frac{e_t}{A_t}\right) \times 100$$

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left(\frac{|e_t|}{A_t}\right) \times 100 .$$

where n = number of periods

$e_t$ = forecast error

$A_t$ = actual value

t = some time period.

The PE measures the ratio of the error to actual and the MPE is the mean or average of the PE's. Just as is true for the ME, the MPE should typically be near zero as positive errors are offset by negative errors. In contrast, absolute values of errors are used in the MAPE; thus, positive and negative errors do not offset each other. Thus, the ME and MPE do not measure error scatter like MAPE.

The MAPE is a sum of the absolute errors for each time period divided by the actual value for the period this sum is divided by the number of periods to obtain a mean value. Then, by convention, this is multiplied by 100 to present it in percentage terms. This is a simple measure permitting comparison across different forecasting models with different time periods and number of observations and weighting all percentage error magnitudes the same. Lower MAPE values are preferred to higher ones because they indicate a forecasting model is producing smaller percentage errors.

Moreover, its interpretation is intuitive. The MAPE indicates, an average, the percentage error a given forecasting model produces for a specified period. Lewis (1982) suggested that a MAPE of less than 10% is considered highly accurate forecasting, a MAPE in the range 10% to 20% is good forecast, a MAPE between 20% and 50% is reasonable forecast and a MAPE of greater than 50% is considered as inaccurate forecasting.

Delurgio (1998) said that when using percentages or ratios, it must be cautious because extremely small denominators can yield extremely high percentages or ratios. This problem is prevalent in forecasting whenever the actual values are very low. Thus, percentage measures have to be monitored for low denominators. Although such low actual values may be outliers and therefore should be adjusted, frequently they are not.

Pankratz (1983) suggested that MAPE is useful for conveying the accuracy of a model to managers or other non-technical users. He also stated that the MAPE roughly suggests the kind of accuracy that can be expected from forecast produced by fitted model. However, the preferred way of conveying forecast accuracy is to derive confidence interval for the forecast values.

## 5.3 Minimum Mean Square Error Forecast

To derive the minimum mean square error forecasts, consider the general stationary ARMA (p, q) model

$$\phi(B)Z_t = \theta(B)a_t \qquad (5.3.1)$$

where $\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p$ and $\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q$ are polynomials of degree p and q in B, $\phi_i$, i = 1, 2, . . . , p and $\theta_j$, j = 1, 2, . . . , q are the autoregressive and moving average parameters of the time series respectively and B is the backward shift operator, that is, $B^j Z_t = Z_{t-j}$ . In the above model, $a_t$ is the white noise or error series with mean zero and variance $\sigma_a^2$ referred to as the error variance.

Because the model is stationary, it can be written in moving average representation,

$$\begin{aligned} Z_t &= \psi(B)a_t \\ &= a_t + \psi_1 a_{t-1} + \psi_2 a_{t-2} + \ldots \end{aligned} \qquad (5.3.2)$$

where

$$\psi(B)=\sum_{j=0}^{\infty}\psi_j B^j=\frac{\theta(B)}{\phi(B)} \qquad (5.3.3)$$

and $\psi_0=1$. For $t = n + L$, we have

$$Z_{n+L}=\sum_{j=0}^{\infty}\psi_j a_{n+L-j}. \qquad (5.3.4)$$

Suppose at time $t = n$ we have the observations $Z_n, Z_{n-1}, Z_{n-2}, \ldots$ and wish to forecast L-step ahead of future value $Z_{n+L}$ as a linear combination of the observations $Z_n, Z_{n-1}, Z_{n-2}, \ldots$. Since $Z_t$ for $t = n, n-1, n-2, \ldots$ can all be written in the form of Equation (5. 3. 2), let the minimum mean square error forecast $\hat{Z}_n(L)$ of $Z_{n+L}$ be

$$\hat{Z}_n(L)=\psi^{*}_{L}a_n+\psi^{*}_{L+1}a_{n-1}+\psi^{*}_{L+2}a_{n-2}+\ldots \qquad (5.3.5)$$

where $\psi^{*}_{j}$ are to be determined. The mean square error of the forecast is

$$E[Z_{n+L}-\hat{Z}_n(L)]^2=\sigma_a^2\sum_{j=0}^{L-1}\psi_j^2+\sigma_a^2\sum_{j=0}^{\infty}[\psi_{L+j}-\psi^{*}_{L+j}]^2,$$

which is easily seen to be minimized when $\psi^{*}_{L+j}=\psi_{L+j}$. Hence,

$$\hat{Z}_n(L)=\psi_L a_n+\psi_{L+1}a_{n-1}+\psi_{L+2}a_{n-2}+\ldots \qquad (5.3.6)$$

By using Equation (5. 3. 4) and the fact that

$$E(a_{n+j}|Z_n,Z_{n-1},\ldots)=\begin{Bmatrix}0, & j>0,\\ a_{n+j}, & j\le 0,\end{Bmatrix}.$$

Then

$$E(Z_{n+L}|Z_n,Z_{n-1},\ldots)=\psi_L a_n+\psi_{L+1}a_{n-1}+\psi_{L+2}a_{n-2}+\ldots.$$

Thus, the minimum mean square error forecast of $Z_{n+L}$ is given by its conditional expectation. That is,

$$\hat{Z}_n(L)=E(Z_{n+L}|Z_n,Z_{n-1},\ldots) \qquad (5.3.7)$$

$\hat{Z}_n(L)$ is the L-step ahead forecast of $Z_{n+L}$ at the forecast origin n.

The forecast error is

$$e_n(L)=Z_{n+L}-\hat{Z}_n(L)=\sum_{j=0}^{L-1}\psi_j a_{n+L-j}. \qquad (5.3.8)$$

Because $E[e_n(L) \mid Z_t, t \le n] = 0$, the forecast is unbiased with variance

$$V[\hat{Z}_n(L)] = V[e_n(L)] = \sigma_a^2 \sum_{j=0}^{L-1} \psi_j^2 . \qquad (5.3.9)$$

If the random shocks $a_t$'s are normally distributed and if it has been estimated an appropriate model with a sufficiently large sample, forecasts from that model are approximately normally distributed. Therefore, confidence intervals around each point forecast using a table of probabilities for standard normal deviations can be constructed. Thus an approximate (1-α)100% confidence interval for the forecast is given by

$$\hat{Z}_n(L) \pm N_{\alpha/2} \left[ 1 + \sum_{j=1}^{L-1} \psi_j^2 \right]^{\frac{1}{2}} \sigma_a \qquad (5.3.10)$$

where $N_{\alpha/2}$ is the standard normal deviate such that $P(Z > Z_{\alpha/2}) = \frac{\alpha}{2}$.

## 5.4 Forecasting Each Data Series with Fitted Models

The adequately fitted models presented in Chapter IV were used in forecasting. In this section, the minimum mean square error forecasts for the fitted models are obtained together with their respective 95% confidence intervals. The forecast values for five years ahead, that is from 2006-2007 to 2010-2011 are also computed and presented together with their respective 95% lower and upper confidence limits for each data series.

### 5.4.1 Base Metal and Ores Export Series

The fitted AR(1) model with an IO at t = 32 and two AO's at t = 40 and t = 44 for base metal and ores export series is given by

$$Z_t = \theta_0 + \frac{1}{1-\phi B}\left[\omega_1 P_t^{(32)} + a_t\right] + \omega_2 P_t^{(40)} + \omega_3 P_t^{(44)}$$

where the estimated parameters are $\theta_0 = 28.076$, $\phi = 0.773$, $\omega_1 = 36.360$, $\omega_2 = 24.547$ and $\omega_3 = -21.453$ with $\sigma_a^2 = 72.063$.

Using the observations $Z_t$ ; t = 1, 2, .....,51 (from 1955-1956 to 2005-2006), the forecast valuues $Z_{52}$(2006-2007), $Z_{53}$(2007-2008), $Z_{54}$(2008-2009), $Z_{55}$(2009-2010) and $Z_{56}$(2010-2011) with their 95% lower and upper confidence limits are to be obtained. The procedure is as follows:

The above fitted AR(1) model with 3 outliers can be rewritten as

$$(1-\phi B)(Z_t - \theta_0) = \omega_1 P_t^{(32)} + (1-\phi B)[\ \omega_2 P_t^{(40)} + \omega_3 P_t^{(44)}\ ] + a_t$$

$$(Z_t - \theta_0) - \phi\ (Z_{t-1} - \theta_0)\ = \omega_1 P_t^{(32)} + (1-\phi B)[\ \omega_2 P_t^{(40)} + \omega_3 P_t^{(44)}\ ] + a_t$$

$$(Z_t - \theta_0)\ = \phi\ (Z_{t-1} - \theta_0) + \omega_1 P_t^{(32)} + (1-\phi B)[\omega_2 P_t^{(40)} + \omega_3 P_t^{(44)}] + a_t$$

and the general form of the forecast equation is

$$\hat{Z}_t(L) = \theta_0 + \phi\ [\hat{Z}_t(L - 1) - \theta_0\ ] + \omega_1 P_t^{(32)} + (1-\phi B)[\ \omega_2 P_t^{(40)} + \omega_3 P_t^{(44)}\ ]$$

$$= \theta_0(1 - \phi) + \phi^L\ Z_t + \omega_1 P_t^{(32)} + \omega_2 P_t^{(40)} + \omega_3 P_t^{(44)} - \phi\omega_2 P_t^{(39)} - \phi\omega_3 P_t^{(43)}\ \ ; L \geq 1 \quad (5.4.1)$$

where L is the lead time.

Then, the $(1 - \alpha)$ 100% forecast limits are computed using the formula in Equation (5.3.10). Thus, the forecast values for $Z_{52}$ (2006 - 2007), $Z_{53}$ (2007- 2008), $Z_{54}$ (2008 - 2009), $Z_{55}$(2009-2010), $Z_{56}$(2010-2011) and their 95% lower and upper forecast limits are obtained as follows:

**Table (5.1)**

**Forecast Values and their 95% Forecast Confidence Limits for Base Metal and Ores Exports Series**

| Time Point | Year | Forecast | Lower Limits | Upper Limits |
|---|---|---|---|---|
| t = 52 | 2006-07 | 29.57 | 13.82 | 45.32 |
| t = 53 | 2007-08 | 29.24 | 9.33 | 49.14 |
| t = 54 | 2008-09 | 28.98 | 6.97 | 50.99 |
| t = 55 | 2009-10 | 28.78 | 5.60 | 51.96 |
| t = 56 | 2010-11 | 28.62 | 4.78 | 52.47 |

For base metal and ores exports series, the forecast values and their 95% forecast confidence limits are plotted in Figure 5.1.

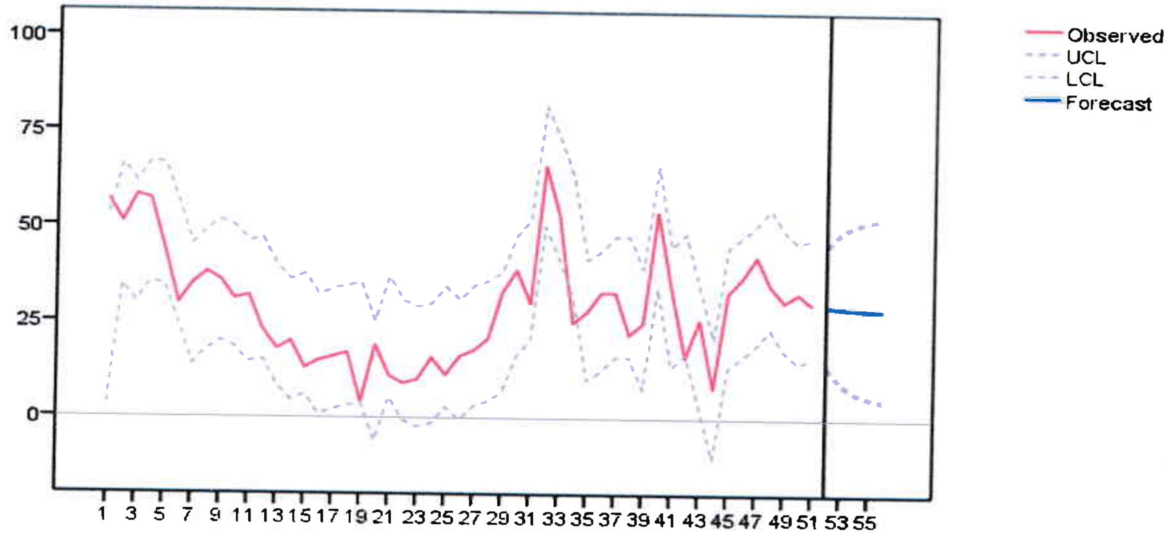


**Figure 5.1** **Plot of Forecast Values and their 95% Confidence Limits for Base Metal and Ores Exports Series**

### 5.4.2 Teak Export Series

For the teak export series, the fitted AR(1) model with an AO at time t = 24 is obtained as

$$Z_t = \theta_0 + \omega_1 P_t^{(24)} + \frac{1}{1-\phi B} a_t$$

where the estimated parameters are $\theta_0$ = 163.201, $\phi$ = 0.932 and $\omega_1$ = 88.921 with $\sigma_a^2$ = 1226.86.

Based on the observations for teak export, $Z_t$ ; t = 1, 2, . . . ,51 (from 1955-1956 to 2005-2006), the forecast values $Z_{52}$(2006-2007), $Z_{53}$(2007-2008), $Z_{54}$(2008-2009), $Z_{55}$(2009-2010) and $Z_{56}$(2010-2011) with their 95% lower and upper confidence limits are to be computed. Then, the procedure is as follows:

The above fitted AR(1) model with an AO outlier can be written as

$$(1 - \phi B)(Z_t - \theta_0) = (1 - \phi B)\,\omega_1 P_t^{(24)} + a_t$$

$$(Z_t - \theta_0) - \phi\,(Z_{t-1} - \theta_0) = (1 - \phi B)\,\omega_1 P_t^{(24)} + a_t$$

$$Z_t - \theta_0 = \phi\,(Z_{t-1} - \theta_0) + (1 - \phi B)\,\omega_1 P_t^{(24)} + a_t$$

$$Z_t = \theta_0 + \phi\,(Z_{t-1} - \theta_0) + (1 - \phi B)\,\omega_1 P_t^{(24)} + a_t$$

and the general form of the forecast equation is

$$\hat{Z}_t(L) = \theta_0 + \phi\,[\hat{Z}_t(L-1) - \theta_0] + (1 - \phi B)\,\omega_1 P_t^{(24)}$$

$$= \theta_0 (1 - \phi) + \phi^L Z_t + \omega_1 P_t^{(24)} - \phi\omega_1 P_t^{(23)} \quad ; L \geq 1 \qquad (5.4.2)$$

where L is the lead time.

Then, the $(1 - \alpha)$ 100% forecast limits are calculated using the formula in Equation (5.3.10). Thus, the forecast values for $Z_{52}$(2006-2007), $Z_{53}$(2007-2008), $Z_{54}$(2008-2009), $Z_{55}$(2009-2010) and $Z_{56}$(2010-2011) as well as their 95% lower and upper confidence limits are obtained as in the following table.

**Table (5.2)**

**Forecast Values and their 95% Forecast Confidence Limits for Teak Export Series**

| Time Point | Year | Forecast | Lower Limits | Upper Limits |
|---|---|---|---|---|
| t = 52 | 2006-07 | 321.51 | 257.28 | 385.75 |
| t = 53 | 2007-08 | 310.80 | 222.98 | 398.62 |
| t = 54 | 2008-09 | 300.81 | 196.74 | 404.88 |
| t = 55 | 2009-10 | 291.50 | 175.14 | 407.86 |
| t = 56 | 2010-11 | 282.82 | 156.74 | 408.90 |

For teak export series, the forecast values and their respective 95% forecast confidence limits are plotted in Figure 5.2.

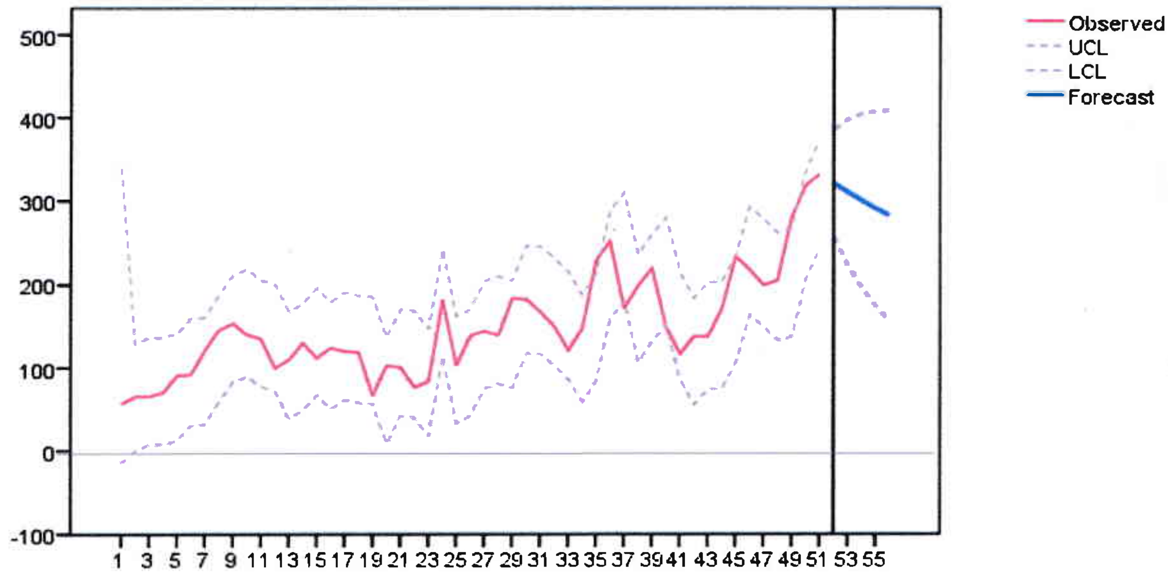


**Figure 5.2 Plot of Forecast Values and their 95% Confidence Limits for Teak Export Series**

### 5.4.3 Wheat Production Series

The fitted AR(1) model with an IO at t = 34 and an AO at t = 29 for wheat production series is given by

$$Z_t = \theta_0 + \frac{1}{1-\phi B}\left[\omega_1 P_t^{(34)} + a_t\right] + \omega_2 P_t^{(29)}$$

where the estimated parameters are $\theta_0$ = 64.839, $\phi$ = 0.936, $\omega_1$ = 86.220 and $\omega_2$ = -49.744 with $\sigma_a^2 = 287.981$.

From the observations on wheat production $Z_t$ ; t = 1, 2, .....,56 (from 1950-1951 to 2005-2006) and the forecast values $Z_{57}$ (2006-2007), $Z_{58}$ (2007-2008), $Z_{59}$ (2008-2009), $Z_{60}$(2009-2010) and $Z_{61}$(2010-2011) with their 95% lower and upper confidence limits are to be obtained. Then, the procedure is as follows:

The above fitted AR(1) model with 2 outliers can be rewritten as

$$(1-\phi B)(Z_t - \theta_0) = \omega_1 P_t^{(34)} + (1-\phi B)[\ \omega_2 P_t^{(29)}] + a_t$$

$$(Z_t - \theta_0) - \phi\ (Z_{t-1} - \theta_0) = \omega_1 P_t^{(34)} + (1-\phi B)[\ \omega_2 P_t^{(29)}] + a_t$$

$$(Z_t - \theta_0) = \phi\ (Z_{t-1} - \theta_0) + \omega_1 P_t^{(34)} + (1-\phi B)[\omega_2 P_t^{(29)}] + a_t$$

$$Z_t = \theta_0 + \phi\ (Z_{t-1} - \theta_0) + \omega_1 P_t^{(34)} + (1-\phi B)[\omega_2 P_t^{(29)}] + a_t$$

and the general form of the forecast equation is

$$\hat{Z}_t(L) = \theta_0 + \phi\,[\hat{Z}_t(L-1) - \theta_0] + \omega_1 P_t^{(34)} + (1-\phi B)[\,\omega_2 P_t^{(29)}]$$

$$= \theta_0(1-\phi) + \phi^L Z_t + \omega_1 P_t^{(34)} + \omega_2 P_t^{(29)} - \phi\omega_2 P_t^{(28)} \;\; ; \;\; L \geq 1 \qquad (5.4.3)$$

where L is the lead time.

Then, the $(1-\alpha)$ 100% forecast limits are computed using the formula in Equation (5.3.10). Thus, the forecast values for $Z_{57}$(2006-2007), $Z_{58}$(2007-2008), $Z_{59}$(2008-2009), $Z_{60}$(2009-2010), $Z_{61}$(2010-2011) and their 95% lower and upper forecast limits are obtained as follows:

**Table (5.3)**

**Forecast Values and their 95% Forecast Confidence Limits for Wheat Production Series**

| Time Point | Year | Forecast | Lower Limits | Upper Limits |
|---|---|---|---|---|
| t = 57 | 2006-07 | 148.0 | 117.2 | 178.9 |
| t = 58 | 2007-08 | 142.7 | 100.5 | 185.0 |
| t = 59 | 2008-09 | 137.8 | 87.6 | 188.0 |
| t = 60 | 2009-10 | 138.5 | 82.3 | 194.7 |
| t = 61 | 2010-11 | 133.8 | 72.8 | 194.8 |

For wheat production series, the forecast values and their 95% forecast confidence limits are plotted in Figure 5.3.

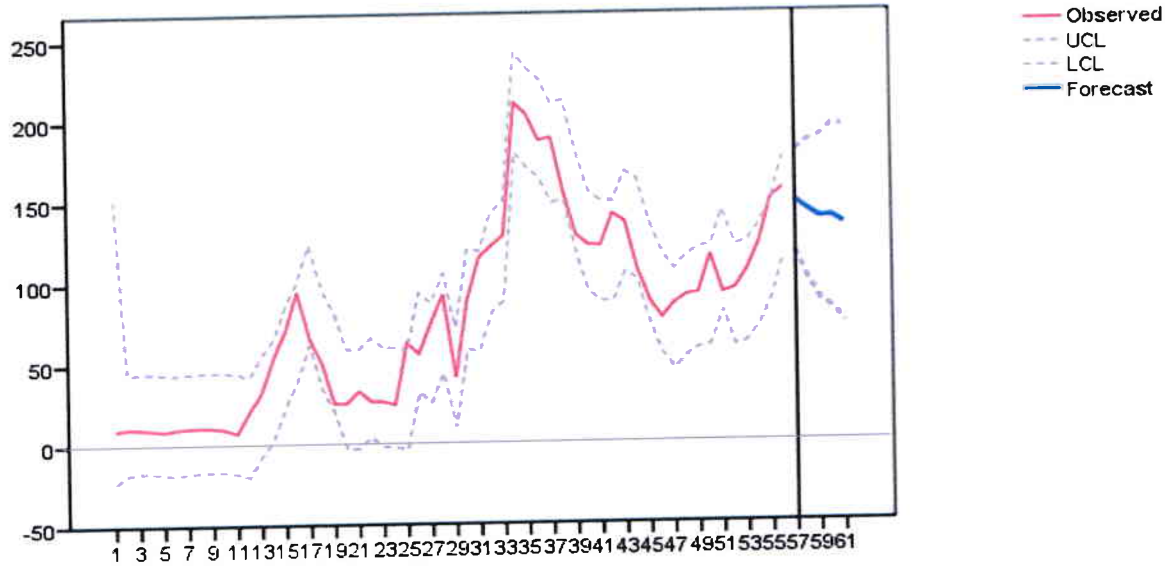


**Figure 5.3** **Plot of Forecast Values and their 95% Confidence Limits for Wheat Production Series**

### 5.4.4 Lablab Bean Production Series

For the lablab bean production series, the fitted AR(1) model with an AO at time t = 30 is obtained as

$$Z_t = \theta_0 + \omega_1 P_t^{(30)} + \frac{1}{1-\phi B} a_t$$

where the estimated parameters are $\theta_0$ = 48.387, $\phi$ = 0.976 and $\omega_1$ = 24.558 with $\sigma_a^2$ = 49.546.

Using the observations on lablab bean production, $Z_t$ ; t = 1, 2, . . . , 56 (from 1950-1951 to 2005-2006) and the forecast values $Z_{57}$(2006-2007), $Z_{58}$(2007-2008), $Z_{59}$(2008-2009), $Z_{60}$(2009-2010) and $Z_{61}$(2010-2011) with their 95% lower and upper confidence limits are to be computed. Then, the procedure is as follows:

The above fitted AR(1) model with an AO outlier can be written as

$$(1 - \phi B)(Z_t - \theta_0) = (1 - \phi B)\, \omega_1 P_t^{(30)} + a_t$$

$$(Z_t - \theta_0) - \phi\, (Z_{t-1} - \theta_0) = (1 - \phi B)\, \omega_1 P_t^{(30)} + a_t$$

$$Z_t - \theta_0 = \phi\, (Z_{t-1} - \theta_0) + (1 - \phi B)\, \omega_1 P_t^{(30)} + a_t$$

$$Z_t = \theta_0 + \phi\, (Z_{t-1} - \theta_0) + (1 - \phi B)\, \omega_1 P_t^{(30)} + a_t$$

and the general form of the forecast equation is

$$\hat{Z}_t(L) = \theta_0 + \phi\,[\hat{Z}_t(L-1) - \theta_0] + (1 - \phi B)\,\omega_1 P_t^{(30)}$$

$$= \theta_0(1 - \phi) + \phi^L Z_t + \omega_1 P_t^{(30)} - \phi\omega_1 P_t^{(29)}\ ; \quad L \geq 1 \qquad (5.4.4)$$

where L is the lead time.

Then, the $(1 - \alpha)$ 100% forecast limits are calculated using the formula in Equation (5.3.10). Thus, the forecast values for $Z_{57}$(2006-2007), $Z_{58}$(2007-2008), $Z_{59}$(2008-2009), $Z_{60}$(2009-2010) and $Z_{61}$(2010-2011) as well as their 95% lower and upper confidence limits are obtained as in the following table.

**Table (5.4)**

**Forecast Values and their 95% Forecast Confidence Limits for Lablab Bean Production Series**

| Time Point | Year | Forecast | Lower Limits | Upper Limits |
|---|---|---|---|---|
| t = 57 | 2006-07 | 89.6 | 77.5 | 101.7 |
| t = 58 | 2007-08 | 88.6 | 71.7 | 105.6 |
| t = 59 | 2008-09 | 87.7 | 67.2 | 108.2 |
| t = 60 | 2009-10 | 86.8 | 63.4 | 110.2 |
| t = 61 | 2010-11 | 85.9 | 60.0 | 111.7 |

For lablab bean production series, the forecast values and their respective 95% forecast confidence limits are plotted in Figure 5.4.

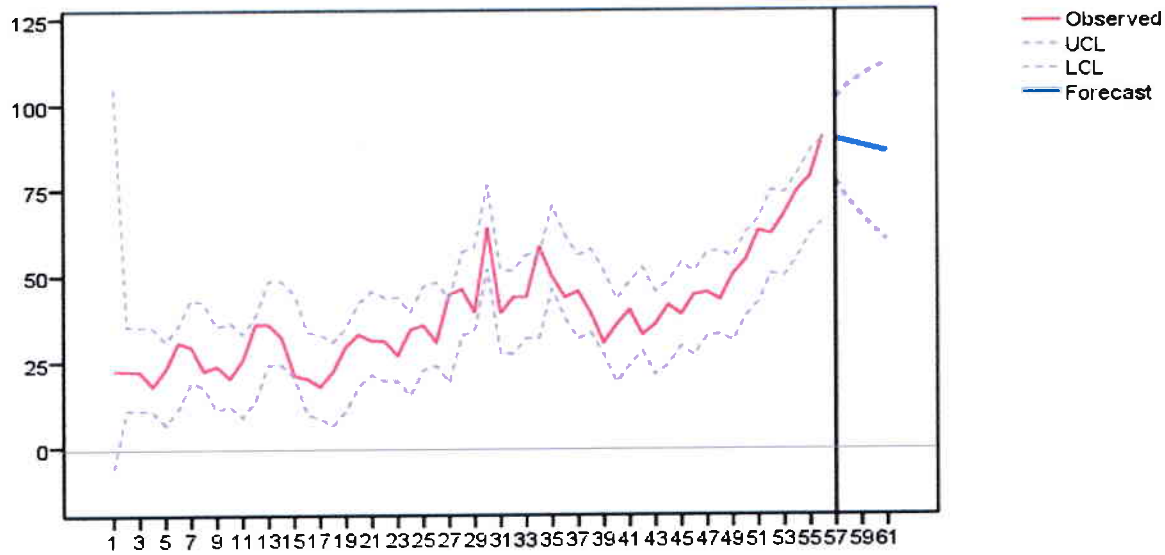


**Figure 5.4 Plot of Forecast Values and their 95% Confidence Limits for Lablab Bean Production Series**

### 5.4.5 Lima Bean Production Series

For the lima bean production series, the fitted AR(1) model with an AO at time t = 14 is given by

$$Z_t = \theta_0 + \omega_1 P_t^{(14)} + \frac{1}{1-\phi B} a_t$$

where the estimated parameters are $\theta_0$ = 4.813, $\phi$ = 0.961 and $\omega_1$ = 3.249 with $\sigma_a^2 = 0.658$.

Based on the observations for lima bean production, $Z_t$ ; t = 1, 2, . . . , 56 (from 1950-1951 to 2005-2006) and the forecast values $Z_{57}$(2006-2007), $Z_{58}$(2007-2008), $Z_{59}$(2008-2009), $Z_{60}$(2009-2010) and $Z_{61}$(2010-2011) with their 95% lower and upper confidence limits are to be computed. Then, the procedure is as follows:

The above fitted AR(1) model with an AO outlier can be written as

$$(1 - \phi B)(Z_t - \theta_0) = (1 - \phi B)\, \omega_1 P_t^{(14)} + a_t$$

$$(Z_t - \theta_0) - \phi\, (Z_{t-1} - \theta_0) = (1 - \phi B)\, \omega_1 P_t^{(14)} + a_t$$

$$Z_t - \theta_0 = \phi\, (Z_{t-1} - \theta_0) + (1 - \phi B)\, \omega_1 P_t^{(14)} + a_t$$

$$Z_t = \theta_0 + \phi\, (Z_{t-1} - \theta_0) + (1 - \phi B)\, \omega_1 P_t^{(14)} + a_t$$

and the general form of the forecast equation is

$$\hat{Z}_t(L) = \theta_0 + \phi\,[\hat{Z}_t(L-1) - \theta_0] + (1 - \phi B)\,\omega_1 P_t^{(14)}$$

$$= \theta_0(1 - \phi) + \phi^L Z_t + \omega_1 P_t^{(14)} - \phi\omega_1 P_t^{(13)} \quad ; \; L \geq 1 \qquad (5.4.5)$$

where L is the lead time.

Then, the $(1 - \alpha)$ 100% forecast limits are calculated using the formula in Equation (5.3.10). Thus, the forecast values for $Z_{57}$(2006-2007), $Z_{58}$(2007-2008), $Z_{59}$(2008-2009), $Z_{60}$(2009-2010) and $Z_{61}$(2010-2011) as well as their 95% lower and upper confidence limits are obtained as in the following table.

**Table (5.5)**

**Forecast Values and their 95% Forecast Confidence Limits for Lima Bean Production Series**

| Time Point | Year | Forecast | Lower Limits | Upper Limits |
|---|---|---|---|---|
| t = 57 | 2006-07 | 7.6 | 6.0 | 9.2 |
| t = 58 | 2007-08 | 7.5 | 5.3 | 9.7 |
| t = 59 | 2008-09 | 7.4 | 4.7 | 10.0 |
| t = 60 | 2009-10 | 7.3 | 4.3 | 10.3 |
| t = 61 | 2010-11 | 7.2 | 3.9 | 10.5 |

For lima bean production series, the forecast values and their respective 95% forecast confidence limits are plotted in Figure 5.5.

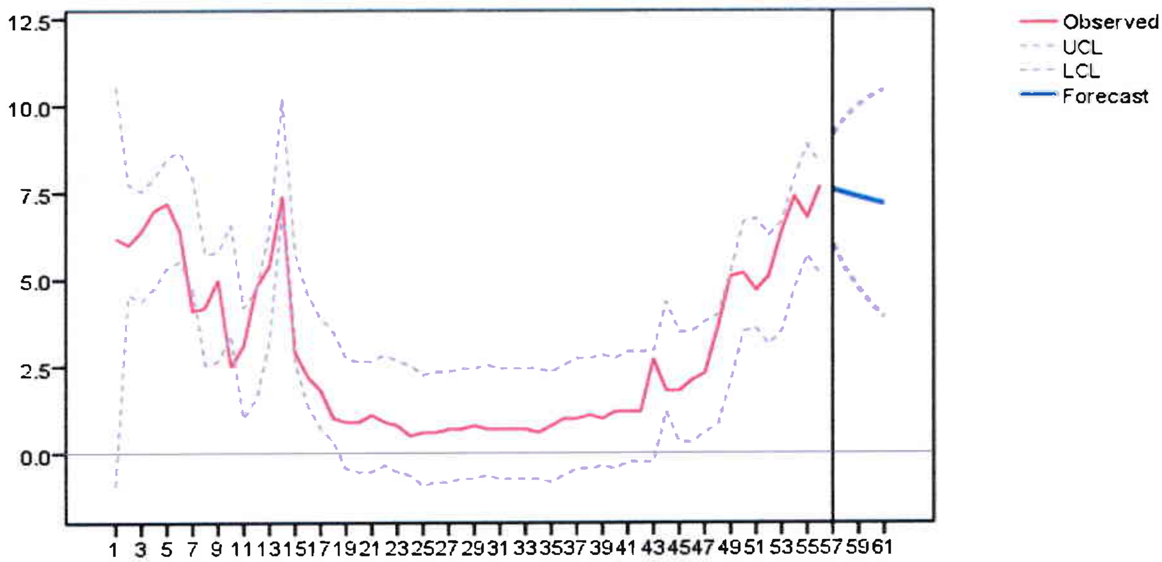


**Figure 5.5 Plot of Forecast Values and their 95% Confidence Limits for Lima Bean Production Series**

### 5.5 Forecast Evaluation for Fitted Models

The mean absolute percentage error (MAPE) values for the models fitted to each series are computed to evaluate the performance of the model fitted to each data series and they are presented in the following table.

**Table (5.6)**

**The MAPE Values for the Fitted Models**

| Series | MAPE |
|---|---|
| 1. Base Metal and Ores Export | 31.014% |
| 2. Teak Export | 18.692% |
| 3. Wheat Production | 31.968% |
| 4. Lablab Bean Production | 15.104% |
| 5. Lima Bean Production | 22.875% |

According to Table (5.6), it can be seen that the MAPE values of the fitted models for teak export series and lablab bean production series are less than 20%. Therefore, the forecast values for these two series are generally considered to be good. Since the MAPE values of the fitted models for base metal and ores export series, wheat production series and lima bean production series lie between 20% and 50%, it can be concluded that forecast values from the fitted models to these three series are generally considered to be reasonable.

The control charts for the fitted model for each data series in this study are presented in Appendix C, Figure C1 to C5. They suggest that the fitted values lie between 95% upper and lower confidence limits and those values fluctuate around with original values for all observed data series in this study. Thus, the fitted model for each data series selected in this study fits the corresponding data sufficiently well and they can give short-term forecasts.

The forecasts for five years ahead (from 2006-2007 to 2010-2011) are also presented in Tables (5.1) to (5.5) and plotted in Figures 5.1 to 5.5 together with their 95% upper and lower prediction limits. It is also observed that all the forecast values fall within the 95% upper and lower confidence limits of the each of the forecast values for all observed series in this study. Therefore, the forecast values are generally considered to be acceptable.

Moreover, the forecast values for one year ahead, that is, for the year 2006-2007 together with actual values for the same period are also presented in the following table since the actual values of the data series selected in this study become available. This was done to compare the actual and forecast values and thereby provide the bases for evaluating the adequacy of the forecast values in presenting data series to which they have been fitted.

**Table (5.7)**

**Forecasts, Actual Values and 95% Upper and Lower Confidence Limits for 2006-2007**

| Series | Forecast | Actual | Lower Limit | Upper Limit |
|---|---|---|---|---|
| 1. Base Metal and Ores Export | 29.57 | 27.0 | 13.82 | 45.32 |
| 2. Teak Export | 321.51 | 347.0 | 257.28 | 385.75 |
| 3. Wheat Production | 148.0 | 140.2 | 177.2 | 178.9 |
| 4. Lablab Bean Production | 89.6 | 93.2 | 77.5 | 101.7 |
| 5. Lima Bean Production | 7.6 | 9.1 | 6.0 | 9.2 |

Results for forecasting presented in Table (5.7) shows that differences between actual and forecast values for the same period are moderate in all data series. Even if there are some differences between actual and forecast values, all forecast values and all actual values fall within the upper and lower confidence limits of each of the forecast values for all data series which are analyzed in this study.

# CHAPTER VI

# CONCLUSION

The analysis of time series data becomes very complex when the included values are not typical of the past or future. These non-typical values are known as outliers, which are very large or small observations that are not indicative of repeating past or future patterns. Outliers in an economic time series include deviations that occur because of unusual events such as policy changes, environmental regulations, economic changes, advertising promotions, supply interruptions, natural disasters, wars, strikes and similar events. However, in many practical situations, the causes and timing of occurring outliers may be unknown.

Outliers may exist in economic time series data and can at least theoretically have harmful effects on their analysis. It is difficult to say that how frequently outliers are occurred and which is the best way to describe them (that is, additive or innovational outlier or others), how serious a threat they pose in practice, and how to handle them or indeed whether anything at all should be done about them.

In practice, the presence of outliers is assumed as indicated in the graph at the start of analysis, additional procedures for detection of outliers and assessment of their possible impacts are very essential. We should stress the importance of adjusting outliers prior to and during the analysis of time series data. Even more emphasis should perhaps be placed on examining and explaining the possible causes of detected outliers in the observed data.

In time series, outliers can take several forms. Fox (1972) early gave the formal definition and classification of outliers in a time series. He proposed a classification of type I and type II outliers based on an autoregressive model. These two types have later be renamed as additive outlier (AO) and innovational outlier (IO) and these two types of outliers are focused in this study.

An AO only affects a single observation of a time series, which is either larger or smaller in value than expected. In contrast, IO affects several subsequent observations in a time series. But, AO's are the most troublesome whereas IO's often have lesser effect than AO's. Moreover, an IO only affects one residual of the series. On the other hand, an AO affects the next consecutive residuals in the series as well. This effect has several consequences for any further analysis of the residuals and the error variance will also be inflated and more alarmingly, the outlier may create a pattern into the residuals (Tolvi, 1998).

The presence of such outliers in a time series can also have substantial effects on the mean and variance of the series. The impact of presence of a single outlier on the series mean and variance is investigated based on AR(1), MA(1) and ARMA(1,1) series using simulated data. It is observed that the larger the magnitude of outlier, the higher the mean and variance are obtained. Besides, it is found that the effect of IO on mean is much more significant for both AR(1) and ARMA(1,1) series but the effect of AO on the mean is much more prominent for MA(1) series. In addition, it is also clear that the effect of IO on the variance is much more obvious than that of AO for both AR(1) and MA(1) as well as ARMA(1,1) cases. Moreover, it can be concluded that the effect of outlier on mean and variance depends on both magnitude and type of outliers.

In time series, outliers can affect on ARMA model identification, estimation and forecasting. First of all, outliers affect the autocorrelation structure of a time series and therefore they also bias the autocorrelation function (ACF) and partial autocorrelation function (PACF). These biases can be severe and they depend on, besides the obvious attributes like the type and magnitude of outliers, also the underlying model and its autocorrelation structure. (Detsuh, Richards, and Swain; 1990). ARMA model identification is traditionally based on the ACFand PACFand will in the presence of outliers therefore be misleading, unless outliers are somehow taken into account (Tolvi, 1998). From the simulated results, it is found that the estimated value of lag-1 autocorrelation $\rho_1$ does not change significantly for IO case of both AR(1) and MA(1) series. However, the estimated value of $\rho_1$ decreases as the value of outlier parameter increases for AO case of both AR(1) and MA(1) series. For ARMA(1,1) series, estimated value of $\rho_1$ decreases as the value of outlier parameter increases for both AO and IO cases. But, the effect of $\rho_1$ is much more obvious in the case of AO than that of IO for ARMA(1,1) series. The same conclusion can be made for the effect of outlier on the PACF since the values of both lag-1 autocorrelation $\rho_1$ and lag-1 partial autocorrelation $\phi_{11}$ are identical for AR(1), MA(1) and ARMA(1,1) series. It can also be concluded that the effect of the presence of a single outlier on the estimates of ACFand PACF depends on both the magnitude and type of outliers for the underlying models of AR(1), MA(1) and ARMA(1,1).

Similarly, outliers bias the estimated ARMA model's parameters and estimated error variances as well. Chang and Tio (1983), Chen and Liu (1993) suggested that the AO outliers cause substantial biases in estimated ARMA parameters, whereas IO's have only minor effects. Ledolter (1989) also stated that outliers inflate the estimated error variance of the series. Based on the simulated results, it is found that estimated parameters (that is, $\phi$ and $\theta$ for AR(1) and MA(1), respectively) do not vary significantly with change in the magnitude of outlier parameter for the case of IO of both AR(1) and MA(1) series. However, it is also clear that the estimated values of parameters $\phi$ and $\theta$ decrease with the increasing magnitude of outlier parameter for the AO case of AR(1) and MA(1) series, respectively. For ARMA(1,1) series, it is also observed that the estimated values of autoregressive parameter $\phi$ do not differ significantly with the change in the magnitude of outlier for both AO and IO cases. But, the estimated values of moving average parameter $\theta$ do not vary obviously with the increasing magnitude of outlier parameter in the IO case whereas the estimated values of moving average parameter $\theta$ decrease noticeably with the increasing magnitude of outlier parameter in the AO case of ARMA(1,1) series.

According to simulated results, it can also be seen that the error variance tends to get overestimated with the larger values of outlier parameter for both AR(1) and MA(1) as well as ARMA(1,1) cases, irrespective type of outlier. This overestimation is more prominent in the AO than that of IO for both AR(1) and MA(1) as well as ARMA(1,1) series. In addition, it is also found that the effect of presence of outlier of either type on error variance is much more than that on the estimated parameters for these series. Besides, it can be concluded that the effect on the estimates of time series parameters as well as the error variance depends on both magnitude and type of outlier.

Outliers can also affect on the model selection criteria including (Akiake's Information Criterion) AIC and BIC (Baye's Information Criterion) as well as they can bias the selection of the model because the computation of AIC and BIC based on the estimated error variance. From the simulation study, it is observed that the increasing values of AIC and BIC are obtained as the values of outlier parameter become larger and the effect of outlier on both AIC and BIC is also much more significant in the case of AO than that of IO for both AR(1) and MA(1) as well as ARMA(1,1) series.

The presence of a single outlier can result in a true AR model being falsely identified as a MA or an ARMA model, and that the identified lag length (p and q) can also be misspecified. The simulated results also suggest that the percentage of correct model selection declines as the value of outlier parameter increases in the AO case whereas it is not true in the IO case of both AR(1) and MA(1) series. It can also be concluded that the effect of AO is more serious for both AR(1) and MA(1) models identification but the effect of IO is not as clear as that of AO.

Besides, the existence of outilers in a time series can affect on the time series model building stages such as model identification, parameter estimation, diagnostic checking and forecasting. Thus, the investigation into the presence of outliers, detection of position of outliers, identification of type of outlier, estimation of parameters in the presence of outliers, assessment of their effects on the analysis and the remedial measures to accommodate the outliers is a crucial aspect of time series analysis.

For detection of outliers, simple statistical tools such as time series plots, frequentcy distributions and simple t-tests can be used. These methods are simple and perhaps useful in some cases, but obviously not sufficient for the wide variety of situations encountered in empirical time series analysis. It is therefore necessary to consider more complicated but effective methods and some of these are presented in the present study. The methods described are classified as likelihood ratio test, influence function method, Q statistics, leave-k-out diagnostics and likelihood displacement for outlier diagnostic.

In Myanmar, many economic time series were affected by events that are planned by decision makers and caused by economic changes, weather conditions, out-of-stock situations, competitors and similar events. This study attempts to detect the timing of the presence of outliers, identifies the type of outliers in selected economic time series of Myanmar and also explains the possible causes of occurrence of such outliers in each observed data series. In the detection and estimation of outliers, this study focuses only on the applications of likelihood ratio test (LRT) using SPSS software and adjustment diagnostics based on error variance (ADV) procedure using STDS software. Then, using SPSS software, ARIMA models with outliers are constructed for selected economic time series and the future values for five years ahead are forecasted based on the fitted models which could be useful in decision-making and planning purposes.

For detection of outliers in selected economic time series of Myanmar, the following ARIMA models with outliers are obtained.

(1) AR(1) model with an IO outlier at t = 32 and two AO outliers at t = 40 , 44 for base metal and ores export series

(2) AR(1) model with an AO outlier at t = 24 for teak export series

(3) AR(1) model with an AO outlier at t = 29 and an IO outlier at t = 34 for wheat production series

(4) AR(1) model with an AO outlier at t = 30 for lablab bean production series

(5) AR(1) model with an AO outlier at t = 14 for lima bean production series

To evaluate the forecast values, the MAPE values for the model fitted to each data series are also computed. Since the MAPE values for all data series are less than 50%, the forecasts obtained from the fitted models are considered to be accurate.

The estimated values and their 95% upper and lower prediction limits as well as the actual values for the studied periods of each observed data series are presented. In addition, the control charts for fitted model to each data series are also presented to see if the estimated 95% confidence interval contains the actual values. They suggest that the fitted values fall between 95% lower and upper confidence limits and those values fluctuate near the original values for all observed data series in this study. The forecast values for five years ahead (from 2006-2007 to 2010-2011) are also computed together with their 95% upper and lower confidence limits. It is also found that all forecast values fall between their 95% upper and lower confidence limits of each of the observed series.

The effect of changes in the economic time series due to special events can be analyzed by statistical outlier analysis in order to provide the timings and the types of outliers to be useful in future planning. In doing so, the application of only one procedure for outlier detection is not enough and it is also suggested to use two or more detection methods to get more satisfactorily results.

This study focuses only on two types of outliers, namely, additive outlier (AO) and innovational outlier (IO) which can occur most often in practical time series. In furture researches, the effects of other types of outliers such as level shift (LS), temporary change (TC), variance change (VC) and reallocation outlier (RO) should be studied by suitable outlier detection procedures.

The detection of outliers can also be extended for seasonal time series models if it is possible for monthly or quarterly time series data. Furthermore, the detection of outliers in multivariate time series should be investigated as a further study. It is also recommended that the outlier detection and model fitting should be studied regularly in order to have better estimates or forecasts.

## REFERENCES


Abraham, B. (1987), Outliers in Time Series, in MacNeill I. B. and G. J. Umphrey (eds.), *Time Series and Econometric Modeling*, D. Reidel Publishing Company, 77-89.

Abraham, B. and G. E. P. Box (1979), Bayesian Analysis of Some Outlier Problems in Time Series, *Biometrika*, 66, 229-236.

Abraham, B. and A. Chuang (1989), Outlier Detection and Time Series Modeling, *Technometrics*, 31, 241-248.

Abraham, B. and J. Ledolter (1983), *Statistical Methods for Forecasting*, John Wiley & Sons, U. S. A.

Akaike, H. (1974), A New Look at the Statistical Model Identification, *IEEE Transactions on Automatic Control*, AC-19, 716-713.

Akaike, H. (1978), A Bayesian Analysis of the Minimum AIC Procedure, *Ann. Inst. Statist. Math.*, 30A, 9-14.

Barnett, V. and T. Lewis (1996), *Outliers in Statistical Data*, Third Edition, New York, John Wiley and Sons Ltd.

Box, G. E. P. and G. M. Jenkins (1976), *Time Series Analysis: Forecasting and Control*, Revised Edition, San Francisco: Holden-Day Inc.

Box, G. E. P., G. M. Jenkins and G. S. Reinsel (1994), *Time Series Analysis: Forecasting and Control*, Third Edition, Englewood Cliff, NJ, Prentice Hall.

Box, G. E. P. and G. C. Taio (1965), A Change in Level of a Non-stationary Time Series, *Biometrika*, 52, 181-192.

Box, G. E. P. and G. C. Taio (1975), Intervention Analysis with Applications to Economic and Environmental Problems, *Journal of the American Statistical Association*, 70, 70-79.

Brockwell, P. J. and R. A. Davis (1991), *Time Series: Theory and Methods*, Second Edition, Springer-Verlag, New York.

Brockwell, P. J. and R. A. Davis (1996), *Introduction to Time Series and Forecasting*, Springer-Verlag, New York.

Brubacher, S. R. and G. T. Wilson (1976), Interpolating Time Series with Applications to Economic and Environmental Problems, *Journal of Applied Statistics*, 25, 107-116.

Bruce, A. G. and R. D. Martin (1989), Leave-k-out Diagnostic for Time Series, *Journal of the Royal Statistical Society*, Ser. B., 51, 363-424.

Bustos, O. H. and V. J. Yohai (1986), Robust Estimates for ARMA Models, *Journal of the American Statistical Association*, 81, 155-168.

Chang, I. and G. C. Taio (1983), Estimation of Time Series Parameters in Presence of Outliers, *Technical Report* No. 8, Statistical Research Center, Graduate School of Business, University of Chicago.

Chang, I., G. C. Taio and C. Chen (1988), Estimation of Time Series Parameters in Presence of Outliers, *Technometrics*, 30, 193-204.

Chatfield, C. (1989), *The Analysis of Time Series: An Introduction*, Fourth Edition, Chapman and Hall, New York.

Chatterjee, S. and A. S. Hadi (1988), *Sensitivity Analysis in Liner Regression*, John Wiley & Sons, New York.

Chen, C. and L. M. Liu (1993), Joint Estimation of Model Parameters and Outlier Effects in Time Series, *Journal of the American Statistical Association*, 88, 284-297.

Chernick, M. R., D. J Downing and D. H. Pike (1982), Detection Outliers in Time Series Data, *Journal of the American Statistical Association*, 77, 743-747.

Cook, R. D. (1986), Assessment of Local Influence, *Journal of the Royal Statistical Society*, Ser. B, 48, 133-169.

Cook, R. D. (1987), Influence Assessment, *Journal of Applied Statistics*, 14, 117-131.

Cook, R. D. and S. Weiberg (1982), Residuals and Influence in Regression, *Monographys on Statistics and Applied Probability*, Champman and Hall, New York.

De Jong, P. and J. R. Penzer (1998), Diagnostic Shocks in Time Series, *Journal of the American Statistical Association*, 93, 796-806.

Delurgio, Stephen A. (1998), *Forecasting Principles and Applications*, McGraw-Hill, U. S. A.

Denby, L. And R. D. Martin (1979), Robust Estimation of the First-Order Autoregressive Parameters, *Journal of the American Statistical Association*, 74, 140-146.

Deutsch, S. J., J. E. Richsrds and J. J. Swain (1990), Effect of a single outlier on ARMA Identification, *Communications in Statistics, Theory and Methods*, 19, 2207-2227.

Fox, A. J. (1972), Outliers in Time Series, *Journal of the Royal Statistical Society*, Ser. B., 34, 350-363.

Glendinning, R. H. (1998), Determining the Order of an ARMA Model from Outlier Contaminated Data, *Communications in Statistics, Theory and Methods*, 27, 13-40.

Hotta, L. K. (1993), The Effect of Additive Outliers ont the Estimates from Aggregated and Disaggregated ARIMA Models, *International Journal of Forecasting*, 9, 85-93.

Justel, A., D. Pena and R. S. Tsay (2001), Detection of Outlier Patches in Autoregressive Time Series, *Statistica Sinica*, 11, 651-673.

Kendall, Sir, M. and J. K. Ord (1990), *Time Series*, Third Edition, Edward Arnold, London.

Ledolter, J. (1989), The Effect of Additive Outliers on the Forecasts from ARIMA Models, *International Journal of Forecasting*, 5, 231-240.

Ledolter, J. (1990), Outlier Diagnostics in Time Series Analysis, *Journal of Time Series Analysis*, 11, 317-324.

Liu, L. M. (2006), *Time Series Analysis and Forecasting*, Second Edition, Scientific Computing Associates Corp.

Ljung, G. M. (1993), On Outlier Detection in Time Series, *Journal of the Royal Statistical Society*, Ser. B., 55, 559-567.

Luceno, A. (1998), Detecting Possibly Non-consecutive Outliers in Industrial Time Series, *Journal of the Royal Statistical Society*, Ser. B., 60, 295-310.

Martin, R. D. (1979), Robust Estimation for Time Series Autoregressions, *Robustness in Statistics*, (eds.), R. L. Launer and G. Wilkinson, Academic Press, Inc., New York, 147-175.

Martin, R. D. and V. J. Yohai (1985), Robustness in Time Series and Estimating ARMA Models, invited paper for *Handbook of Statistics, Volume V: Time Series in the Time Domain*, edited by Hannan, Krishnainhand Rao, North-Holland, 119-155.

Martin, R. D. and J. E. Zeh (1977), Determining the Character of Time Series Outliers, *Proc. Econ. Statist. Sect. Amer. Statist. Ass.*, 818-823.

Mills, T. C. (1994), *Time Series Techniques for Economists*, Cambridge University Press, New York.

Murihead, C. R. (1986), Distinguishing Outlier Types in Time Series, *Journal of the Royal Statistical Society*, Ser. B., 48, 39-47.

McCulloh, R. E. and R. S. Tsay (1994), Bayesain Analysis of Autoregressive Time Series Via the Gibbs Sampler, *Journal of Time Series Analysis*, 15, 235-250.

Nelson, C. R. (1973), *Applied Time Series Analysis for Management Forecasting*, Holden-Day, Inc., U. S. A.

Pankratz, A. (1983), *Forecasting with Univariate Box-Jenkins Models: Concepts and Cases*, John Wiley and Sons, New York, U. S. A.

Pena, D. (1987), Measuring the Importance of Outliers in ARIMA Models, *New Perspectives in Theoretical and Applied Statistics*, John Wiley and Sons, New York, U. S. A., 109-118.

Schmid. W. (1986), The Multiple Outlier Problem in Time Series Analysis, *Austral. J. Statist*. 28 (3), 400-413.

Schmid. W.(1989), Identification of Type I Outlier in an Autoregressive Model, *Statistics*, 20, 531-546.

Schmid. W.(1990), Discussion of a LR Test for Detecting Outliers in Time Series Data, *Statistics and Decisions*, 8, 271-294.

Schmid. W.(1996), An Outlier Test for Time Series Based on a Two-sided Predictor, *Journal of Time Series Analysis*, 17, 497-510.

Shibata, R. (1976), Selection of the Order of an Autoregressive Model by Akaike's Information Criterion, *Biometrika*, 63, 117-126.

Soe Win (2004), Adjustment Diagnostic for Outliers in Time Series, Unpublished Ph. D Thesis, University of Hyderabad, India.

Taio, G. C. (1985), Autoregressive Moving Average Models, Intervention Problems and Outlier Detection in Time Series, Invited Paper for *Handbook of Statistics, Volume V: Time Series in Time Domain*, North-Holland, 85-118.

Tolvi, J. (1998), Outliers in Time Series, *Research Report*, 76, Department of Economics, University of Turku.

Tolvi, J. (2001), Outliers in Eleven Finnish Macroeconomic Time Series, *Finnish Economic Papers,* Volume 14(1), Pages 14-32, Spring, Finnish Economic Association.

Tsay, R. S. (1986), Time Series Model Specification in the Presence of Outliers, *Journal of the American Statistical Association*, 81, 132-141.

Tsay, R. S. (1988), Outliers, Level Shifts and Variance Changes in Time Series, *Journal of Forecasting*, 7, 1-20.

Tsay, R. S., D. Pena and A. E. Pankratz (2000), Outliers in Multivariate Time Series, *Biometrika*, 87, 789-804.

Trivez, F. J. (1993), Level Shifts, Temporary Changes and Forecasting, *Journal of Forecasting*, 14, 543-550.

Wei, W. W. S (1990), *Time Series Analysis: Univariate and Multivariate Methods*, Addison-Wesley Publishing Co., U. S. A.

Wu, L. S. Y., R.M. Hosking and N. Ravishanker (1993), Reallocation Outlier in Time Series, *Journal of Applied Statistics*, 42, 301-313.

**Data Sources**

Central Statistical Organization (CSO), *Selected Monthly Economic Indicators*, Various Issues, Ministry of National Planning and Economic Development, Myanmar.

Central Statistical Organization (CSO), *Statistical Yearbook*, Various Issues, Ministry of National Planning and Economic Development, Myanmar.

Ministry of National Planning and Economic Development, *Review of Financial, Economic and Social Conditions*, Various Issues, Myanmar.

Ministry of National Planning and Finance, *Report to the Pyithu Hluttaw on the Financial, Economic and Social Conditions*, Various Issues, Myanmar.

Ministry of Planning and Finance, *Report to the People by the Government of Union of Burma on the Financial, Economic and Social Conditions*, Various Issues, Myanmar.

# APPENDICES

# APPENDIX A

## Akaike's Information Criterion (AIC)

Assume that a statistical model of M parameters is fitted to data. To assess the quality of the model fitting., Akaike (1973) introduced an information criterion. The criterion has been called Akaike's Information Criterion (AIC) in the literature and is defined as

$$AIC = n \ln \hat{\sigma}_a^2 + 2M$$

where M is the number of the parameters in the fitted model, n is the number of residuals that can be calculated from the series and $\hat{\sigma}_a^2$ is the maximum likelihood estimate of the error variance $\sigma_a^2$. The optimal order of the model is chosen by the value of M, which is a function of p and q, so that AIC is minimum.

## Bayesian Information Criterion (BIC)

Shibata (1976) has shown that the AIC criterion tends to overestimate the order of autoregression. More recently, Akaike (1978) has developed a Bayesian extension of the minimum AIC procedure, called Bayesian Information Criterion (BIC), which takes the following form:

$$BIC = n \ln \hat{\sigma}_a^2 - (n - M) \ln\left(1 - \frac{M}{n}\right) + M \ln n + M_z \ln\left[\frac{\left(\frac{\hat{\sigma}_z^2}{\hat{\sigma}_a^2} - 1\right)}{M}\right]$$

where M is the number of the parameters in the fitted model, n is the number of residuals that can be calculated from the series, $\hat{\sigma}_a^2$ is the maximum likelihood estimate of the error variance $\sigma_a^2$ and $\hat{\sigma}_z^2$ is the sample variance of the series. Through a simulation study Akaike (1978) has claimed that the BIC is less likely to overestimate the order of the autoregression.

## APPENDIX B

### Table (B. 1)

### Selected Economic Time Series with Outliers

| Years | Base Metal and Ores Export (thousand metric ton) | Teak Export (thousand cubic ton) | Wheat Production (thousand metric ton) | Lablab Bean Production (thousand metric ton) | Lima Bean Production (thousand metric ton) |
|---|---|---|---|---|---|
| 1950-51 | | | 9.5 | 22.5 | 6.2 |
| 1951-52 | | | 10.6 | 22.4 | 6 |
| 1952-53 | | | 10.4 | 22.2 | 6.4 |
| 1953-54 | | | 9.6 | 18 | 7 |
| 1954-55 | | | 8.8 | 22.9 | 7.2 |
| 1955-56 | 57 | 57 | 10.1 | 30.7 | 6.4 |
| 1956-57 | 51 | 66 | 10.7 | 29.4 | 4.1 |
| 1957-58 | 58 | 66 | 10.9 | 22.4 | 4.2 |
| 1958-59 | 57 | 71 | 10.7 | 23.8 | 5 |
| 1959-60 | 44 | 91 | 9.7 | 20.3 | 2.5 |
| 1960-61 | 30 | 92 | 7.3 | 25.6 | 3.1 |
| 1961-62 | 35 | 121 | 20.7 | 36 | 4.8 |
| 1962-63 | 38 | 146 | 31.9 | 36 | 5.4 |
| 1963-64 | 36 | 154 | 53.4 | 32.2 | 7.4 |
| 1964-65 | 31 | 140 | 70.6 | 21 | 2.9 |
| 1965-66 | 32 | 135 | 94.8 | 20.1 | 2.2 |
| 1966-67 | 23 | 100 | 65.7 | 17.8 | 1.8 |
| 1967-68 | 18 | 110 | 50.3 | 22 | 1 |
| 1968-69 | 20 | 130 | 25.4 | 29.5 | 0.9 |
| 1969-70 | 13 | 112 | 25.4 | 33 | 0.9 |
| 1970-71 | 15 | 124 | 32.9 | 31.1 | 1.1 |
| 1971-72 | 16 | 120 | 26.5 | 31.1 | 0.9 |
| 1972-73 | 17 | 119 | 26.3 | 26.8 | 0.8 |
| 1973-74 | 4 | 67 | 24.3 | 34.4 | 0.5 |
| 1974-75 | 19 | 103 | 62.6 | 35.6 | 0.6 |
| 1975-76 | 11 | 101 | 55.7 | 30.7 | 0.6 |
| 1976-77 | 9 | 77 | 75.2 | 44.6 | 0.7 |
| 1977-78 | 10 | 84 | **92.2** | **46.1** | 0.7 |

| | | | | | |
|---|---|---|---|---|---|
| 1978-79 | 15.7 | 182.6 | 41 | 39.4 | 0.8 |
| 1979-80 | 11.3 | 103 | 89.4 | 63.8 | 0.7 |
| 1980-81 | 16.2 | 138.6 | 114.9 | 39.1 | 0.7 |
| 1981-82 | 17.7 | 144.9 | 122 | 43.8 | 0.7 |
| 1982-83 | 20.8 | 139.2 | 128 | 43.8 | 0.7 |
| 1983-84 | 33 | 184 | 210.2 | 58.5 | 0.6 |
| 1984-85 | 38.7 | 183.1 | 203 | 49.6 | 0.8 |
| 1985-86 | 30.1 | 168.1 | 186.9 | 43.7 | 1 |
| 1986-87 | 66 | 150.7 | 188.7 | 45.5 | 1 |
| 1987-88 | 53 | 121 | 154.4 | 38.9 | 1.1 |
| 1988-89 | 25 | 148 | 128.1 | 30.5 | 1 |
| 1989-90 | 28 | 229 | 122.2 | 35.7 | 1.2 |
| 1990-91 | 33 | 252 | 121.5 | 40 | 1.2 |
| 1991-92 | 33 | 172 | 141.1 | 32.8 | 1.2 |
| 1992-93 | 22 | 199 | 136.4 | 35.9 | 2.7 |
| 1993-94 | 25 | 220 | 106.9 | 41.4 | 1.8 |
| 1994-95 | 54 | 150 | 87.7 | 38.7 | 1.8 |
| 1995-96 | 34 | 117 | 76.7 | 44.5 | 2.1 |
| 1996-97 | 16 | 138 | 85.4 | 45.1 | 2.3 |
| 1997-98 | 26 | 138 | 90.7 | 43 | 3.6 |
| 1998-99 | 8 | 172 | 92 | 50.3 | 5.1 |
| 1999-00 | 33 | 234 | 115.3 | 54.4 | 5.2 |
| 2000-01 | 37 | 218 | 92.1 | 63 | 4.7 |
| 2001-02 | 42.5 | 200 | 94.4 | 62.1 | 5.1 |
| 2002-03 | 35 | 205.6 | 105.7 | 67.9 | 6.4 |
| 2003-04 | 30.8 | 281.1 | 122.4 | 74.7 | 7.4 |
| 2004-05 | 32.8 | 319.2 | 150 | 78.7 | 6.8 |
| 2005-06 | 29.7 | 333.1 | 156.2 | 90.6 | 7.7 |

Source: Report to the People (1964-1965 to 1979-1980)

Report to the Phyithu Hluttaw (1971-1972 to 1988-1989)

Review of Financial, Economic and Social Conditions (1989-1990 to 1997-1998)

Selected Monthly Economic Indicators (1955-2008)

**Table (B. 2)**

**Estimated Values and Their 95% Lower and Upper Confidence Limits of the Fitted Model for Base Metal and Ores Export Series: 1955-1956 to 2005-2006**

| Year | Estimates | Lower Limit | Upper Limit |
|---|---|---|---|
| 1955-56 | 30.4332 | 4.3484 | 56.5179 |
| 1956-57 | 52.6756 | 38.4111 | 66.9402 |
| 1957-58 | 47.6523 | 33.3877 | 61.9168 |
| 1958-59 | 53.5129 | 39.2483 | 67.7774 |
| 1959-60 | 52.6756 | 38.4111 | 66.9402 |
| 1960-61 | 41.7917 | 27.5271 | 56.0563 |
| 1961-62 | 30.0705 | 15.8059 | 44.3351 |
| 1962-63 | 34.2566 | 19.9921 | 48.5212 |
| 1963-64 | 36.7683 | 22.5038 | 51.0329 |
| 1964-65 | 35.0939 | 20.8293 | 49.3584 |
| 1965-66 | 30.9077 | 16.6432 | 45.1723 |
| 1966-67 | 31.7450 | 17.4804 | 46.0095 |
| 1967-68 | 24.2099 | 9.9453 | 38.4745 |
| 1968-69 | 20.0238 | 5.7592 | 34.2884 |
| 1969-70 | 21.6982 | 7.4337 | 35.9628 |
| 1970-71 | 15.8376 | 1.5731 | 30.1022 |
| 1971-72 | 17.5121 | 3.2475 | 31.7767 |
| 1972-73 | 18.3493 | 4.0848 | 32.6139 |
| 1973-74 | 19.1866 | 4.9220 | 33.4511 |
| 1974-75 | 8.3026 | -5.9620 | 22.5672 |
| 1975-76 | 20.8610 | 6.5964 | 35.1256 |
| 1976-77 | 14.1632 | -0.1014 | 28.4278 |
| 1977-78 | 12.4887 | -1.7758 | 26.7533 |
| 1978-79 | 13.3260 | -0.9386 | 27.5905 |
| 1979-80 | 18.0982 | 3.8336 | 32.3627 |
| 1980-81 | 14.4144 | 0.1498 | 28.6789 |
| 1981-82 | 18.5168 | 4.2522 | 32.7813 |

| | | | |
|---|---|---|---|
| 1982-83 | 19.7726 | 5.5080 | 34.0372 |
| 1983-84 | 22.3680 | 8.1034 | 36.6326 |
| 1984-85 | 32.5822 | 18.3176 | 46.8468 |
| 1985-86 | 37.3544 | 23.0898 | 51.6189 |
| 1986-87 | 66.0000 | 51.7354 | 80.2646 |
| 1987-88 | 60.2107 | 45.9461 | 74.4752 |
| 1988-89 | 25.0000 | 10.7354 | 39.2646 |
| 1989-90 | 25.8844 | 11.6198 | 40.1489 |
| 1990-91 | 28.3961 | 14.1315 | 42.6606 |
| 1991-92 | 32.5822 | 18.3176 | 46.8468 |
| 1992-93 | 32.5822 | 18.3176 | 46.8468 |
| 1993-94 | 23.3727 | 9.1081 | 37.6373 |
| 1994-95 | 50.3708 | 36.1062 | 64.6354 |
| 1995-96 | 29.6632 | 15.3987 | 43.9278 |
| 1996-97 | 33.4194 | 19.1548 | 47.6840 |
| 1997-98 | 18.3493 | 4.0848 | 32.6139 |
| 1998-99 | 5.2078 | -9.0568 | 19.4724 |
| 1999-00 | 29.6634 | 15.3989 | 43.9280 |
| 2000-01 | 32.5822 | 18.3176 | 46.8468 |
| 2001-02 | 35.9311 | 21.6665 | 50.1957 |
| 2002-03 | 40.5358 | 26.2713 | 54.8004 |
| 2003-04 | 34.2566 | 19.9921 | 48.5212 |
| 2004-05 | 30.7403 | 16.4757 | 45.0049 |
| 2005-06 | 32.5822 | 18.3176 | 46.8468 |

**Table (B. 3)**

**Estimated Values and Their 95% Lower and Upper Confidence Limits of the Fitted Model for Teak Export Series: 1955-1956 to 2005-2006**

| Year | Estimates | Lower Limit | Upper Limit |
|---|---|---|---|
| 1955-56 | 163.2008 | -14.4451 | 340.8466 |
| 1956-57 | 64.1856 | -0.0484 | 128.4197 |
| 1957-58 | 72.5767 | 8.3426 | 136.8107 |
| 1958-59 | 72.5767 | 8.3426 | 136.8107 |
| 1959-60 | 77.2384 | 13.0043 | 141.4724 |
| 1960-61 | 95.8852 | 31.6511 | 160.1192 |
| 1961-62 | 96.8175 | 32.5835 | 161.0515 |
| 1962-63 | 123.8553 | 59.6213 | 188.0894 |
| 1963-64 | 147.1638 | 82.9298 | 211.3979 |
| 1964-65 | 154.6225 | 90.3885 | 218.8566 |
| 1965-66 | 141.5698 | 77.3357 | 205.8038 |
| 1966-67 | 136.9081 | 72.6740 | 201.1421 |
| 1967-68 | 104.2762 | 40.0422 | 168.5102 |
| 1968-69 | 113.5996 | 49.3656 | 177.8336 |
| 1969-70 | 132.2464 | 68.0124 | 196.4804 |
| 1970-71 | 115.4643 | 51.2302 | 179.6983 |
| 1971-72 | 126.6524 | 62.4183 | 190.8864 |
| 1972-73 | 122.9230 | 58.6890 | 187.1570 |
| 1973-74 | 121.9907 | 57.7566 | 186.2247 |
| 1974-75 | 73.5090 | 9.2750 | 137.7430 |
| 1975-76 | 107.0732 | 42.8392 | 171.3073 |
| 1976-77 | 105.2085 | 40.9745 | 169.4426 |
| 1977-78 | 82.8324 | 18.5984 | 147.0664 |
| 1978-79 | 178.2795 | 114.0454 | 242.5135 |
| 1979-80 | 98.3832 | 34.1492 | 162.6172 |
| 1980-81 | 107.0732 | 42.8392 | 171.3073 |
| 1981-82 | 140.2645 | 76.0305 | 204.4985 |

| | | | |
|---|---|---|---|
| 1982-83 | 146.1382 | 81.9042 | 210.3723 |
| 1983-84 | 140.8239 | 76.5899 | 205.0579 |
| 1984-85 | 182.5927 | 118.3587 | 246.8267 |
| 1985-86 | 181.7536 | 117.5196 | 245.9876 |
| 1986-87 | 167.7685 | 103.5345 | 232.0025 |
| 1987-88 | 151.5458 | 87.3118 | 215.7798 |
| 1988-89 | 123.8553 | 59.6213 | 188.0894 |
| 1989-90 | 149.0285 | 84.7945 | 213.2625 |
| 1990-91 | 224.5480 | 160.3139 | 288.7820 |
| 1991-92 | 245.9918 | 181.7577 | 310.2258 |
| 1992-93 | 171.4046 | 107.1706 | 235.6387 |
| 1993-94 | 196.5778 | 132.3438 | 260.8118 |
| 1994-95 | 216.1569 | 151.9229 | 280.3910 |
| 1995-96 | 150.8932 | 86.6591 | 215.1272 |
| 1996-97 | 120.1260 | 55.8919 | 184.3600 |
| 1997-98 | 139.7051 | 75.4711 | 203.9391 |
| 1998-99 | 139.7051 | 75.4711 | 203.9391 |
| 1999-00 | 171.4046 | 107.1706 | 235.6387 |
| 2000-01 | 229.2097 | 164.9756 | 293.4437 |
| 2001-02 | 214.2922 | 150.0582 | 278.5263 |
| 2002-03 | 197.5101 | 133.2761 | 261.7442 |
| 2003-04 | 202.7312 | 138.4972 | 266.9653 |
| 2004-05 | 273.1229 | 208.8888 | 337.3569 |
| 2005-06 | 308.4585 | 244.2245 | 372.6925 |

**Table (B. 4)**

**Estimated Values and Their 95% Lower and Upper Confidence Limits of the Fitted Model for Wheat Production Series: 1955-1956 to 2005-2006**

| Year | Estimates | Lower Limit | Upper Limit |
|---|---|---|---|
| 1950-51 | 64.8385 | -23.0417 | 152.7188 |
| 1951-52 | 13.0219 | -17.8282 | 43.8720 |
| 1952-53 | 14.0519 | -16.7982 | 44.9019 |
| 1953-54 | 13.8646 | -16.9855 | 44.7147 |
| 1954-55 | 13.1155 | -17.7346 | 43.9656 |
| 1955-56 | 12.3664 | -18.4837 | 43.2165 |
| 1956-57 | 13.5837 | -17.2664 | 44.4338 |
| 1957-58 | 14.1455 | -16.7046 | 44.9956 |
| 1958-59 | 14.3328 | -16.5173 | 45.1829 |
| 1959-60 | 14.1455 | -16.7046 | 44.9956 |
| 1960-61 | 13.2091 | -17.6410 | 44.0592 |
| 1961-62 | 10.9619 | -19.8882 | 41.8120 |
| 1962-63 | 23.5091 | -7.3410 | 54.3592 |
| 1963-64 | 33.9963 | 3.1462 | 64.8464 |
| 1964-65 | 54.1280 | 23.2779 | 84.9781 |
| 1965-66 | 70.2333 | 39.3832 | 101.0834 |
| 1966-67 | 92.8932 | 62.0431 | 123.7433 |
| 1967-68 | 65.6452 | 34.7951 | 96.4953 |
| 1968-69 | 51.2253 | 20.3752 | 82.0754 |
| 1969-70 | 27.9100 | -2.9401 | 58.7600 |
| 1970-71 | 27.9100 | -2.9401 | 58.7600 |
| 1971-72 | 34.9326 | 4.0825 | 65.7827 |
| 1972-73 | 28.9399 | -1.9101 | 59.7900 |
| 1973-74 | 28.7527 | -2.0974 | 59.6028 |
| 1974-75 | 26.8800 | -3.9701 | 57.7300 |
| 1975-76 | 62.7425 | 31.8924 | 93.5926 |
| 1976-77 | 56.2816 | 25.4315 | 87.1317 |
| 1977-78 | 74.5406 | 43.6905 | 105.3907 |
| 1978-79 | 40.7147 | 9.8646 | 71.5648 |

| | | | |
|---|---|---|---|
| 1979-80 | 89.0953 | 58.2452 | 119.9454 |
| 1980-81 | 87.8369 | 56.9868 | 118.6869 |
| 1981-82 | 111.7140 | 80.8639 | 142.5641 |
| 1982-83 | 118.3621 | 87.5120 | 149.2122 |
| 1983-84 | 210.2000 | 179.3499 | 241.0501 |
| 1984-85 | 200.9489 | 170.0988 | 231.7990 |
| 1985-86 | 194.2071 | 163.3570 | 225.0572 |
| 1986-87 | 179.1317 | 148.2817 | 209.9818 |
| 1987-88 | 180.8172 | 149.9671 | 211.6673 |
| 1988-89 | 148.7001 | 117.8500 | 179.5502 |
| 1989-90 | 124.0739 | 93.2238 | 154.9240 |
| 1990-91 | 118.5494 | 87.6993 | 149.3995 |
| 1991-92 | 117.8939 | 87.0439 | 148.7440 |
| 1992-93 | 136.2466 | 105.3965 | 167.0966 |
| 1993-94 | 131.8457 | 100.9956 | 162.6958 |
| 1994-95 | 104.2231 | 73.3730 | 135.0732 |
| 1995-96 | 86.2450 | 55.3950 | 117.0951 |
| 1996-97 | 75.9451 | 45.0950 | 106.7952 |
| 1997-98 | 84.0914 | 53.2413 | 114.9415 |
| 1998-99 | 89.0541 | 58.2040 | 119.9042 |
| 1999-00 | 90.2714 | 59.4213 | 121.1215 |
| 2000-01 | 112.0885 | 81.2384 | 142.9386 |
| 2001-02 | 90.3650 | 59.5149 | 121.2151 |
| 2002-03 | 92.5186 | 61.6686 | 123.3687 |
| 2003-04 | 103.0995 | 72.2494 | 133.9496 |
| 2004-05 | 118.7367 | 87.8866 | 149.5868 |
| 2005-06 | 144.5801 | 113.7301 | 175.4302 |

**Table (B. 5)**

**Estimated Values and Their 95% Lower and Upper Confidence Limits of the Fitted Model for Lablab Bean Production Series: 1955-1956 to 2005-2006**

| Year | Estimates | Lower Limit | Upper Limit |
|---|---|---|---|
| 1950-51 | 49.3871 | -6.2338 | 105.0079 |
| 1951-52 | 23.1473 | 11.0163 | 35.2782 |
| 1952-53 | 23.0497 | 10.9187 | 35.1806 |
| 1953-54 | 22.8545 | 10.7235 | 34.9854 |
| 1954-55 | 18.7556 | 6.6247 | 30.8866 |
| 1955-56 | 23.5376 | 11.4067 | 35.6686 |
| 1956-57 | 31.1499 | 19.0189 | 43.2808 |
| 1957-58 | 29.8812 | 17.7502 | 42.0121 |
| 1958-59 | 23.0497 | 10.9187 | 35.1806 |
| 1959-60 | 24.4160 | 12.2850 | 36.5469 |
| 1960-61 | 21.0002 | 8.8693 | 33.1312 |
| 1961-62 | 26.1726 | 14.0417 | 38.3036 |
| 1962-63 | 36.3223 | 24.1913 | 48.4532 |
| 1963-64 | 36.3223 | 24.1913 | 48.4532 |
| 1964-65 | 32.6138 | 20.4828 | 44.7447 |
| 1965-66 | 21.6834 | 9.5524 | 33.8143 |
| 1966-67 | 20.8050 | 8.6741 | 32.9360 |
| 1967-68 | 18.5604 | 6.4295 | 30.6914 |
| 1968-69 | 22.6593 | 10.5284 | 34.7903 |
| 1969-70 | 29.9788 | 17.8478 | 42.1097 |
| 1970-71 | 33.3945 | 21.2636 | 45.5254 |
| 1971-72 | 31.5402 | 19.4093 | 43.6712 |
| 1972-73 | 31.5402 | 19.4093 | 43.6712 |
| 1973-74 | 27.3438 | 15.2128 | 39.4747 |
| 1974-75 | 34.7608 | 22.6298 | 46.8917 |
| 1975-76 | 35.9319 | 23.8010 | 48.0629 |
| 1976-77 | 31.1499 | 19.0189 | 43.2808 |
| 1977-78 | 44.7152 | 32.5843 | 56.8462 |
| 1978-79 | 46.1791 | 34.0482 | 58.3101 |

| | | | |
|---|---|---|---|
| 1979-80 | 64.1980 | 52.0670 | 76.3289 |
| 1980-81 | 39.4867 | 27.3557 | 51.6176 |
| 1981-82 | 39.3476 | 27.2167 | 51.4786 |
| 1982-83 | 43.9345 | 31.8036 | 56.0654 |
| 1983-84 | 43.9345 | 31.8036 | 56.0654 |
| 1984-85 | 58.2806 | 46.1497 | 70.4116 |
| 1985-86 | 49.5949 | 37.4639 | 61.7258 |
| 1986-87 | 43.8369 | 31.7060 | 55.9679 |
| 1987-88 | 45.5936 | 33.4626 | 57.7245 |
| 1988-89 | 39.1525 | 27.0215 | 51.2834 |
| 1989-90 | 30.9547 | 18.8237 | 43.0856 |
| 1990-91 | 36.0295 | 23.8986 | 48.1604 |
| 1991-92 | 40.2260 | 28.0950 | 52.3569 |
| 1992-93 | 33.1993 | 21.0684 | 45.3303 |
| 1993-94 | 36.2247 | 24.0937 | 48.3556 |
| 1994-95 | 41.5923 | 29.4613 | 53.7232 |
| 1995-96 | 38.9573 | 26.8263 | 51.0882 |
| 1996-97 | 44.6176 | 32.4867 | 56.7486 |
| 1997-98 | 45.2032 | 33.0723 | 57.3342 |
| 1998-99 | 43.1538 | 31.0228 | 55.2847 |
| 1999-00 | 50.2780 | 38.1471 | 62.4090 |
| 2000-01 | 54.2793 | 42.1484 | 66.4103 |
| 2001-02 | 62.6723 | 50.5413 | 74.8032 |
| 2002-03 | 61.7940 | 49.6630 | 73.9249 |
| 2003-04 | 67.4543 | 55.3234 | 79.5853 |
| 2004-05 | 74.0906 | 61.9597 | 86.2216 |
| 2005-06 | 77.9943 | 65.8634 | 90.1253 |

**Table (B. 6)**

**Estimated Values and Their 95% Lower and Upper Confidence Limits of the Fitted Model for Lima Bean Production Series: 1950-1951 to 2005-2006**

| Year | Estimates | Lower Limit | Upper Limit |
|---|---|---|---|
| 1950-51 | 6.6717 | 0.9046 | 12.4387 |
| 1951-52 | 6.2121 | 4.9131 | 7.5111 |
| 1952-53 | 6.0173 | 4.7182 | 7.3163 |
| 1953-54 | 6.4070 | 5.1080 | 7.7060 |
| 1954-55 | 6.9916 | 5.6925 | 8.2906 |
| 1955-56 | 7.1864 | 5.8874 | 8.4854 |
| 1956-57 | 4.1000 | 2.8009 | 5.3990 |
| 1957-58 | 4.1661 | 2.8671 | 5.4651 |
| 1958-59 | 4.2635 | 2.9645 | 5.5625 |
| 1959-60 | 2.5000 | 1.2009 | 3.7990 |
| 1960-61 | 2.6072 | 1.3082 | 3.9062 |
| 1961-62 | 3.1918 | 1.8928 | 4.4908 |
| 1962-63 | 4.8481 | 3.5491 | 6.1471 |
| 1963-64 | 8.6813 | 7.3823 | 9.9804 |
| 1964-65 | 4.2161 | 2.9171 | 5.5151 |
| 1965-66 | 2.9969 | 1.6979 | 4.2959 |
| 1966-67 | 2.3149 | 1.0159 | 3.6139 |
| 1967-68 | 1.9252 | 0.6262 | 3.2242 |
| 1968-69 | 1.1458 | -0.1533 | 2.4448 |
| 1969-70 | 1.0483 | -0.2507 | 2.3473 |
| 1970-71 | 1.0483 | -0.2507 | 2.3473 |
| 1971-72 | 1.2432 | -0.0558 | 2.5422 |
| 1972-73 | 1.0483 | -0.2507 | 2.3473 |
| 1973-74 | 0.9509 | -0.3481 | 2.2499 |
| 1974-75 | 0.6586 | -0.6404 | 1.9576 |
| 1975-76 | 0.7560 | -0.5430 | 2.0550 |
| 1976-77 | 0.7560 | -0.5430 | 2.0550 |
| 1977-78 | 0.8535 | -0.4456 | 2.1525 |
| 1978-79 | 0.8535 | -0.4456 | 2.1525 |

| | | | |
|---|---|---|---|
| 1979-80 | 0.9509 | -0.3481 | 2.2499 |
| 1980-81 | 0.8535 | -0.4456 | 2.1525 |
| 1981-82 | 0.8535 | -0.4456 | 2.1525 |
| 1982-83 | 0.8535 | -0.4456 | 2.1525 |
| 1983-84 | 0.8535 | -0.4456 | 2.1525 |
| 1984-85 | 0.7560 | -0.5430 | 2.0550 |
| 1985-86 | 0.9509 | -0.3481 | 2.2499 |
| 1986-87 | 1.1458 | -0.1533 | 2.4448 |
| 1987-88 | 1.1458 | -0.1533 | 2.4448 |
| 1988-89 | 1.2432 | -0.0558 | 2.5422 |
| 1989-90 | 1.1458 | -0.1533 | 2.4448 |
| 1990-91 | 1.3406 | 0.0416 | 2.6396 |
| 1991-92 | 1.3406 | 0.0416 | 2.6396 |
| 1992-93 | 1.3406 | 0.0416 | 2.6396 |
| 1993-94 | 2.8021 | 1.5031 | 4.1011 |
| 1994-95 | 1.9252 | 0.6262 | 3.2242 |
| 1995-96 | 1.9252 | 0.6262 | 3.2242 |
| 1996-97 | 2.2175 | 0.9185 | 3.5165 |
| 1997-98 | 2.4123 | 1.1133 | 3.7114 |
| 1998-99 | 3.6789 | 2.3799 | 4.9780 |
| 1999-00 | 5.1404 | 3.8414 | 6.4394 |
| 2000-01 | 5.2378 | 3.9388 | 6.5368 |
| 2001-02 | 4.7507 | 3.4517 | 6.0497 |
| 2002-03 | 5.1404 | 3.8414 | 6.4394 |
| 2003-04 | 6.4070 | 5.1080 | 7.7060 |
| 2004-05 | 7.3813 | 6.0823 | 8.6803 |
| 2005-06 | 6.7967 | 5.4977 | 8.0957 |

## APPENDIX C

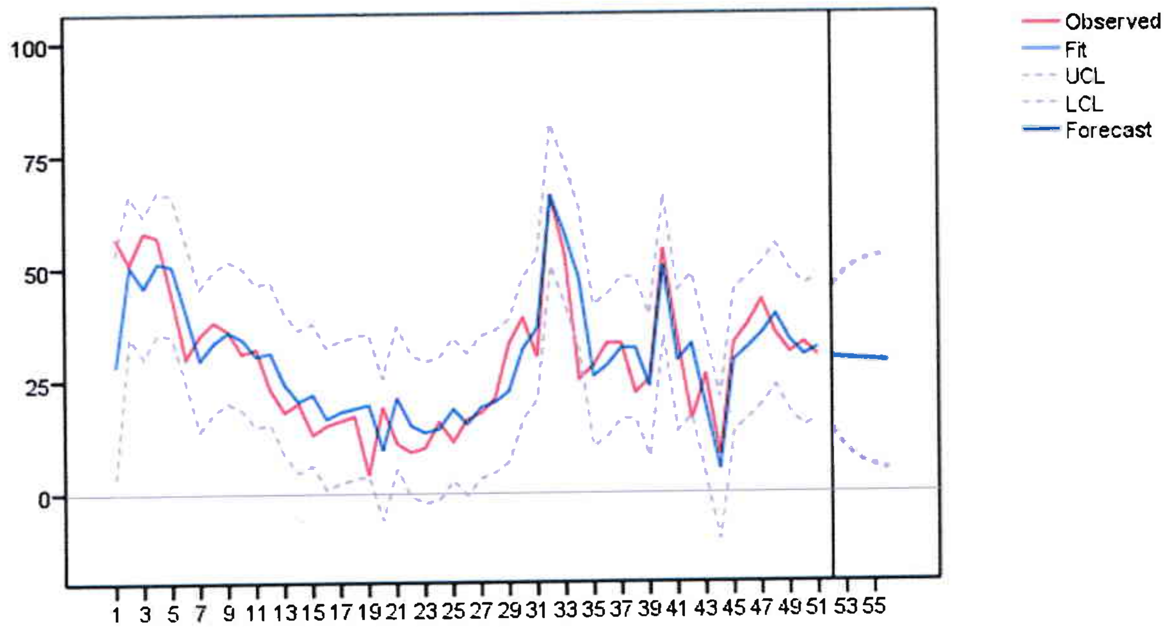


**Figure C1 Control Chart of Fitted Model for Base Metal and Ores Exports Series**

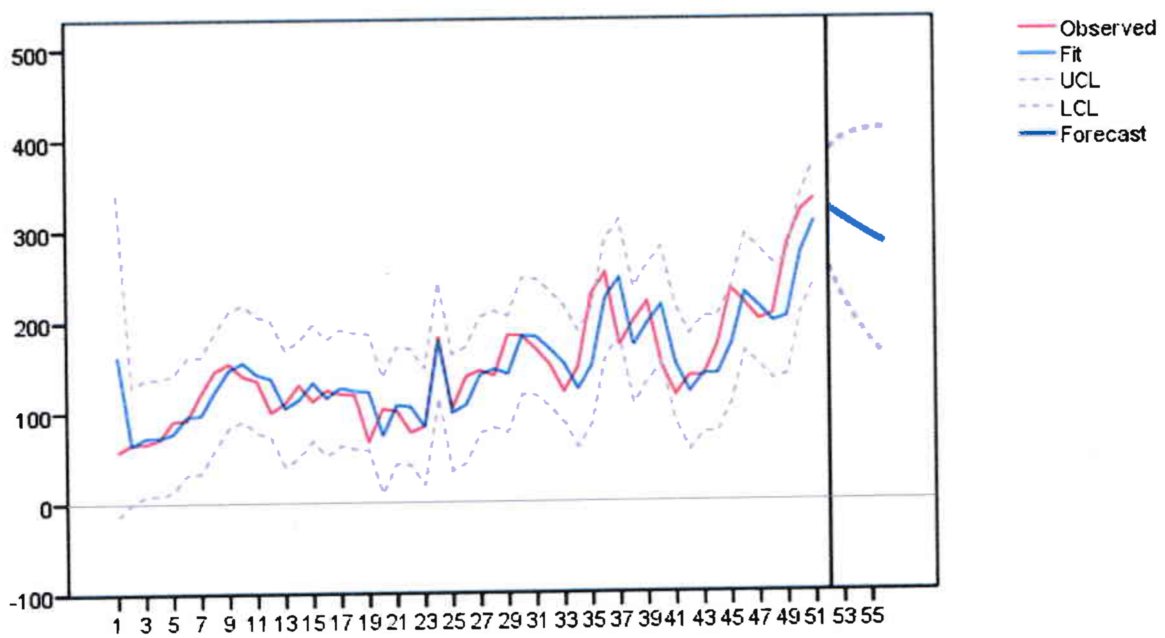


**Figure C2 Control Chart of Fitted Model for Teak Export Series**

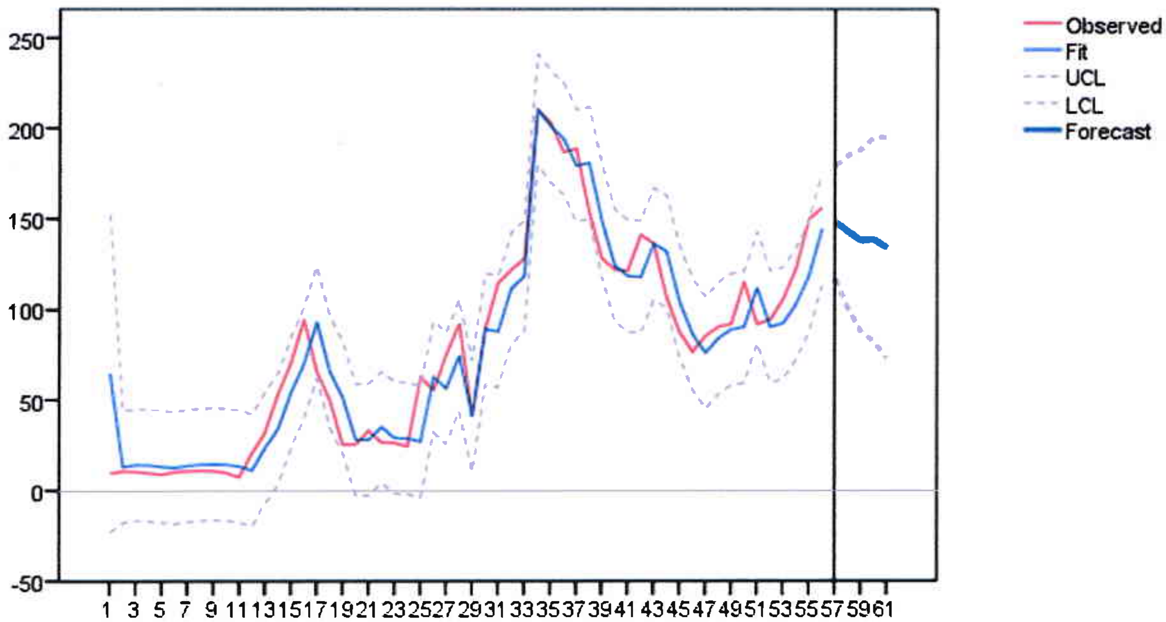


**Figure C3 Control Chart of Fitted Model for Wheat Production Series**

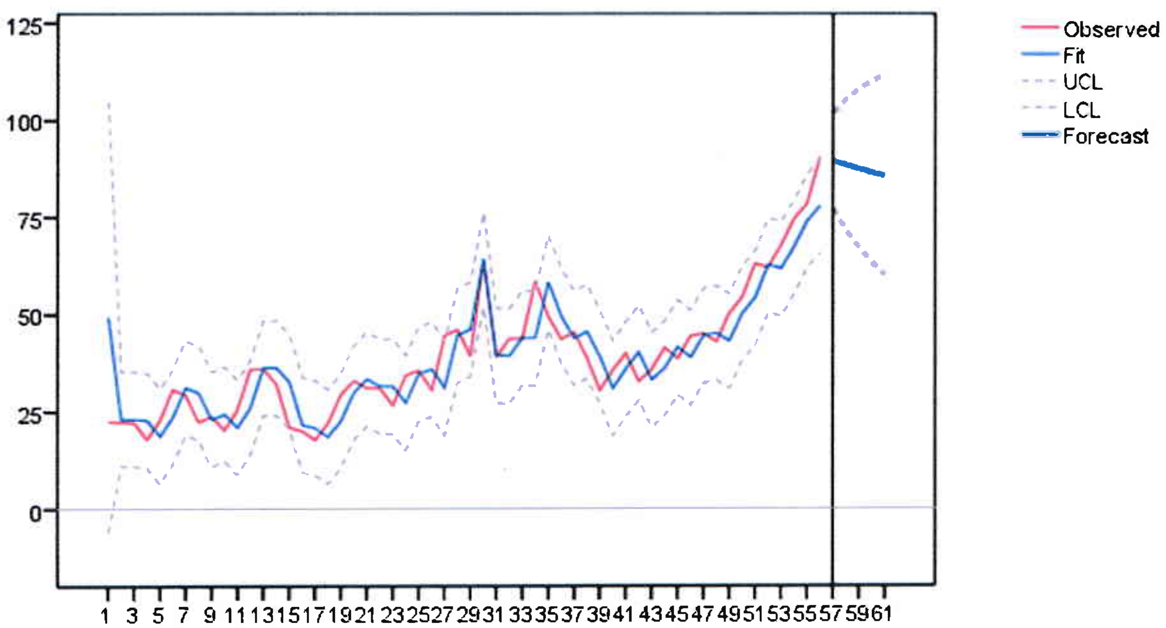


**Figure C4 Control Chart of Fitted Model for Lablab Bean Production Series**

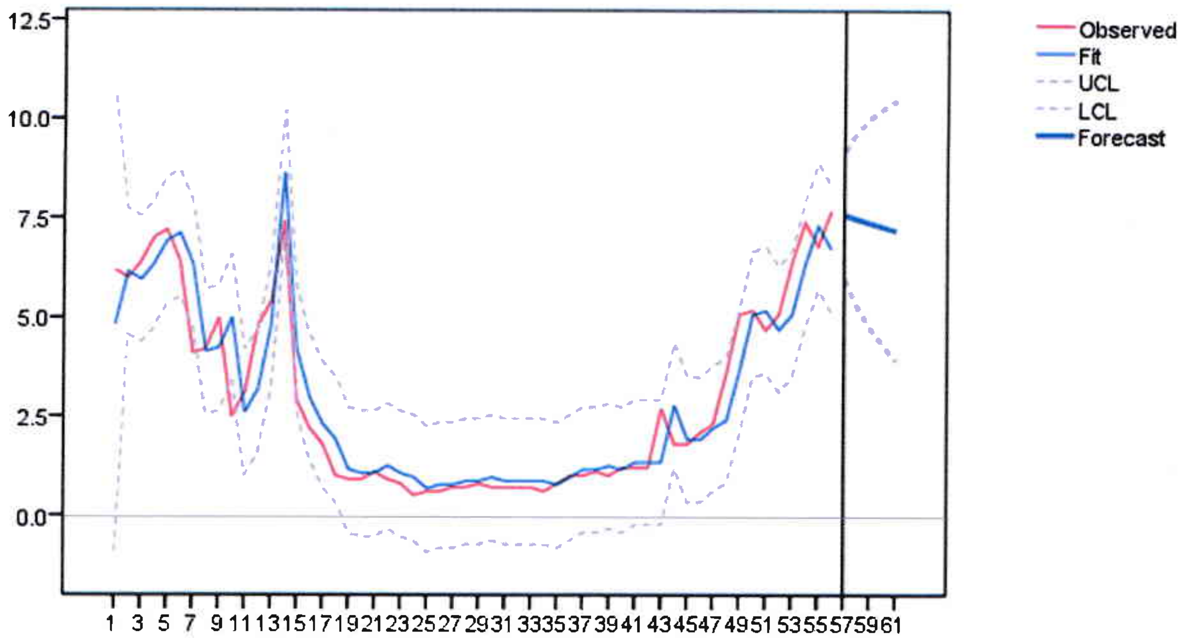


**Figure C5 Control Chart of Fitted Model for Lima Bean Production Series**